

## Course 4: Restoration of Lost Corners Study Guide

**COURSE DESCRIPTION:**

This course consists of four videos, some reading, and three exercises, on the "Restoration of Lost Corners". The legal, mathematical, and practical applications of the methods of proportioning, as found in the Manual of Surveying Instructions, are presented. Students will be able to address what corners control in most situations, how to proportion properly, what legal principles are involved when proportioning, and how to deal with the latitudinal curve. A lengthy discussion of convergence and curvature in the PLSS is also included.

**COURSE OBJECTIVES:**

Upon completion of this course, students will be able to:

- Define the three corner conditions listed in the Manual of Surveying Instructions
- Describe, identify applicability, and compute proportions using all methods
- Demonstrate an understanding of curvature in the PLSS









**COURSE INSTRUCTOR(S):**

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 Ron Scherler, Bureau of Land Management

**VIDEO LECTURE TITLE:**

Restoration of Lost Corners – Part 3 (55 minutes)

ICON LEGEND

 WEB COURSE	 EXERCISE	 DIAGRAM	 READING ASSIGNMENT	 PROBLEM	 HANDOUT	 2009 BLM MANUAL	 QUIZ
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## RESTORATION OF LOST CORNERS – PART 3

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### Introduction

Welcome back, this is Video Lecture 3 of Restoration of Lost Corners here in the CFedS courses. Ron just finished talking to you about curvature and convergency. I know that this is new to most of us that have not been born and raised in the BLM.

I learned quite a bit in that session as well because I had learned other ways. They come out the same but I had learned other ways to do these things. So very educational. I hope that you found it useful. And of course the advantage of being on DVD is that you can play it back over and over again. Well now we are going to completely shift gears. All of that in the previous discussion was all about single proportioning and we ended with single proportions on lines that are curved.

Well now it is time to move on to double proportions. And so let's take a look at some things there. Double proportions are described in Section 7-8 of the Manual and I will read that to you in a minute.

It applies to section corners and township corners but only where corners were truly established in the four way establishment. That is a problem with completion surveys that you will see and it requires the use of cardinal equivalents and true bearing.

Now true bearing of course we keep talking about it and the Manual expects that you will always be on true bearing and we expect you to always be on true bearing. Let's go to 7-8, I realize that you have read this as part of your assignment, but I want to focus on just a few sentences here. 7-8 the term double proportion of measurement is applied to a new measurement made between four known corners to each on intersecting meridional latitudinal lines for the purpose of relating the intersection to both.

Now frankly that is a terribly written sentence as to what is really going on here because it makes it sound like this is an intersection of lines in the sense of a bearing-bearing intersection which it is not.

### Double Proportions

- Described in 7-8 of Manual
- Apply to section corners and township corners
- Only corners with true 4-way establishment
- Requires the use of "cardinal equivalents" and True Bearing

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Now a lot of surveyors believe that a double proportion is a bearing bearing intersection. That is not true. And I even know some state tests that have been conducted for surveyors where that is that is the answer they are looking for that is a bearing bearing intersection. That is totally inappropriate.

In effect, it says here by double proportionate measurement the record directions are disregarded. The directions are going to be disregarded. Now how is that and why is that? That is an interesting question. Now if go down in that second paragraph down about two-thirds of the way you will see a sentence that is in italics.

And it is in italics for a reason and you might want to highlight it yellow in your Manual because in my opinion it is the most misunderstood sentence in the entire book. Lengths of proportioned lines are comparable only when reduced to their **cardinal equivalents**. They are comparable only when they are reduced to their cardinal equivalents. Now what in the world does that mean?

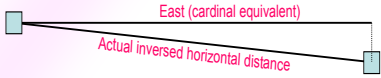
Well let's take a look at the slide. What is a cardinal equivalent? Lengths of proportioned lines are not comparable until reduced to their cardinal equivalents.

Cardinal is one of the four points of the compass and what so we are doing, it is just the same computation as when you reduce for a slope distance except we are in a horizontal plain here.

So what we are doing in that drawing at the bottom of the slide is that we are looking down on a situation and maybe the actual inversed horizontal distance is on you know say looking at that guessing maybe north 88 degrees west, but the cardinal equivalent is not the distance along that line between the two corners, the

### What is a Cardinal Equivalent?

- *"Lengths of proportioned lines are not comparable until reduced to their cardinal equivalents"*
- Cardinal is one of the four points of the compass
- Like reducing a slope distance:



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cardinal equivalent is what is the difference in eastings. In other words what is the distance between those but only in the cardinal direction of that line. So that would be east.

So you know if this was using the slope distance analogy, if that was north 88 west there, then you would take the cosine of two degrees times whatever your inverse distance is and that would give you the cardinal equivalent. That is what a cardinal equivalent is and it is required in the Public Land System for a number of things and in particular for double proportions.

So now let's think about this for a minute because the cardinal equivalent suddenly brings up some issues that you maybe you've already thought about but you know most surveyors and even a lot of software that are supposedly automatically double proportions that sort of thing.

It doesn't use the correct distances, it does not use the cardinal equivalents. It uses the actual inverse distance, which is not what the Manual says to do. Now I recognize one of the problems in the Manual, see figures 7-1 and 7-2, you've got those diagrams there.

That middle diagram that looks like a gun telescope or cross hairs you know that is very poorly drawn as far as how this is supposed to really relate. Because that is how people think that this is some kind of a bearing-bearing intersection or some other solution.

What you have to understand is that line AB and line CD in that drawing is cardinal. They are already reduced to their cardinal equivalency. So you see most surveyors are using the wrong distances in a double proportion, they are using the actual distance from point to point and that is not what the book says and in fact it specifically said that they are only comparable when reduced to their cardinal equivalence.

Really when you think about it, there is some logic in that. If you've got a line running at this bearing and then another one due east and how far is it from one end of the line to the other? Well do you add this length and this length? But you see that is not really how far they are and we are not even interested in the

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distance inverse between the two ends, we are interested in the cardinal only.

That way the corners to the east and west only control the east west position their north south relationship the east west corners, their north south relationship to each other is not a factor.

That is why the Manual said that in effect the record directions or bearings are disregarded. We remove them so we bring everything to apples. See otherwise when you do a double proportion you've got in the example we will have here in a few minutes, you have apples, oranges, you know, watermelons, and lemons. You've got things at all different kinds of bearings and adding and subtracting and dividing those is not a correct way to do a proportion because it starts to weight one line over the other where it shouldn't.

And so to do a cardinal equivalent, we reduce everything to cardinal equivalents so that we can add and subtract and multiply and divide those numbers and everything is on the common ground, that being cardinal directions.

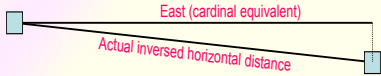
And then so obviously, flipping those corners to the north and south, they will control the north south position of the corner but the east west relationship for the north and south corners will not affect that. That's what's going on here.

Now cardinal equivalents then. A very important question we want to ask then is, how does your basis of bearings affect the cardinal equivalent?

You see this is why you've got to be on astronomic which the Manual assumes you are all the time because if you are on any other basis of bearing, you will change the answer. You will change the location of your lost corner.

### Cardinal Equivalents

- How does your "basis of bearings" affect the cardinal equivalent?
- The Manual assumes you are on an astronomic basis at all times
- If you are any other basis, you will change the answer and location of the lost corner.
- Do you see how it "disregarded the bearings"?



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Let's just think about that for a minute. If you have the example there that I used you know the north 88 west, whatever the cosine is 2 degrees times half a mile, you know you can do that yourself. And then say you are on different basis of bearings. You are on one where you think the inverse distance actually is east. So now there is no cardinal reduction it's just the half mile.

Well compare those two and you will see that those are, you know, I don't know I don't have my calculator here in front of me but you know what I'm saying? That you did that you know it's probably going to make a foot or two difference in what number you are going to proportion against, of course you are going to do that four times. You're going to make that error four times because your lines are at different bearings.

So there is no way to predict how far you will be off. I know that in the sample problem we'll do here in a few minutes if you were I think its two degrees off for your basis of bearings, we just did a what if.

It will actually affect, you'll move the corner a little over three feet. So you know this is worth thinking about and realizing that your basis of bearings is absolutely critical at all times and not that I want to say well there are a few times that you can ignore it, I mean obviously with a single proportion on a straight line, you're going to ignore it?

But your whole project as a CFedS and frankly as any kind of surveyor doing should be on true bearing and so what that does is allows you to do the correct cardinal reductions or cardinal equivalents with these projects. So that is how we disregarded the bearings.

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So now let's run through a double proportion process, just list the steps and then I'll show you how to do that. The first step is that we are going to reduce all of the record distances to cardinal equivalents. That is what **ce** stands for there. And then you are going to reduce all of your measured distances to cardinal equivalents.

Because you see to compare these distances in math, they have to all be cardinal equivalents. We will then compute the proportion north and south and the east west and this gives us two different, you know the north south will give you a northing and the east west will give you an easting and then where those intersect or in other words cardinal moves, you're going to have an answer.

Now what you have to, that will give you the position of your corner, but what you have to remember is that if you reduce the record to cardinal and the measured to cardinal then the answer you get is cardinal. So you are not going to take that cardinal distance that you get and run up the line at some bearing, you are going to go due north south or due east west with that number. So we can establish the lost point.

I'll show you how to do that quite easily especially with coordinates and then from that coordinate you will have to inverse back out to your controlling corners to get what your bearings actually are. So that is kind of the step by step process of a double proportion. I realize that this is new to many of you that is because this is just one of those things that just has never been taught or properly taught even in many of our colleges and universities. So let's do a sample problem here.

### Double Proportion Process

1. Reduce record distances to CE
2. Reduce measured distances to CE
3. Compute proportions (N/S and E/W)
4. Remember: answer is a cardinal number
5. Establish lost point
6. Inverse bearings to controlling corners

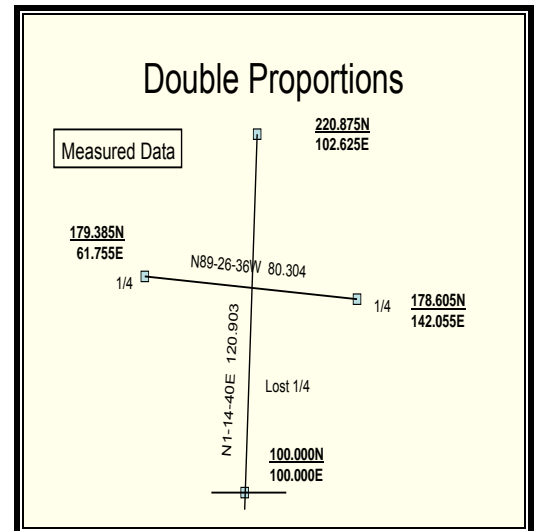
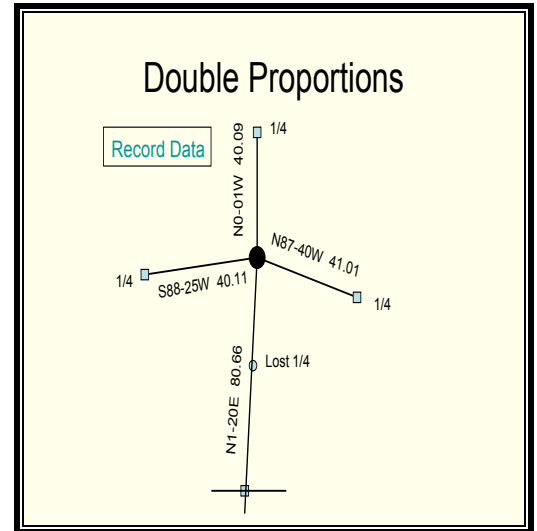
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This screen shows you the record data of a double proportion and you can see there is some what we call some heavy bearings in there this is obviously a section corner that had been found by a previous retracement.

It would be very rare that you would have the original survey show this at this kind of heavy bearings. That would all have been usually in most parts of the country within the 21 Minute Rule. But you can see that three of the four lines coming into this lost section corner are what we call heavy, you know they are outside of the limits but so how did this happen?

It is because of a retracement. So we have to take every one of those numbers based on their corresponding bearing and reduce them to cardinal equivalents.

Now the next slide is the measured data. And what I have done is just given you coordinates and just assume that you have either traversed through those or GPS'd it whatever this is just simple coordinates in chains, all right? But what do we do with these? So let's do the north south computation first.





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As you can on this slide, we are going to do two separate single proportions. Now that we have looked at our record and measured data and discussed the process here, now we are going to do two separate single proportions. Remember that we are going to do them as cardinal equivalents. Everything in here will be cardinal.

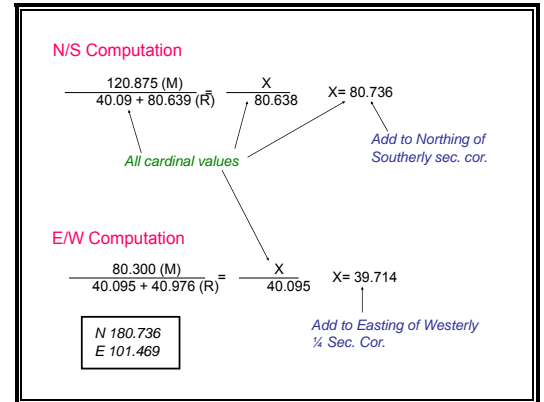
Let us look at this north-south computation first. 120.875 in the measured. Where did I come up with that? That is measured data. Let us go back to the measured data slide. Remember, that if you are on the correct basis of bearings which you must be and you are using coordinates - these are in chains. They can be in anything. Where these corners are based on the proper basis of bearings that the difference of the northing there, and the northing there is a cardinal equivalent. Because the northing difference only will be right up at due north line.

So, we don't have to do any computations on the measured data if you are using coordinates to come up with a computation. That is where the 120.875 comes from for the measured because we can subtract between our coordinates to get that. Now the record, we are going to have to do by hand if you will. Where did we come up with these numbers here? Let us take a look at that.

On the record data, the north/south we are doing. The first line – the line to the north, it is only one minute from cardinal, so you could take the co sine from one minute times 40.09 and it will not come out to anything other than 40.09 unless you carry it way out. My point is there is no cardinal equivalent there. The cardinal equivalent for that line is 40.09. But, on the southerly portion the mile that we are going to do here is one degree twenty minutes from cardinal.

In order to get the cardinal equivalent for the record on this line is we are going to have to take the co sine of one degree twenty minutes and multiply that times 80.66 and that is going to give us these two numbers here. Forty point zero nine (40.09) because there was no cardinal equivalent and 80.638. The computation is in this case because the measured is on top.

The measured over the record for the whole mile and then X is to I am going to solve X for the mile. Actually, it is a mile and half over here. A mile and a half and I am going to solve for the



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southerly mile. Now when you do this computation, you come up with 80.736. Let us remember that is all of the numbers we used in here were cardinal and then that answer is cardinal. To be able to – this makes it simple when working with coordinates because you can take that number and add it to the northing of the southerly section corner. That is because I solved for the southerly portion, so let's see what that looks like.

I would take the 100 that is down here and I am going to add to it the 80.736 that I just computed. That is a northing only - a cardinal number. What that is giving me is a coordinate for somewhere up here wherever that section corners is going to be. It is lost so we are computing it now. It is going to give me that coordinate. 100 and 80.736 is the northing of the lost section corner. Similarly, on the east/west computation, let us run through that.

I have got in the measured 80.300, so where did that come from. The exact same thing we did last time. I am going to take easting of my easterly corner over here 142.055 and I am going to subtract from it 61.755 because the difference of those easting's is a cardinal equivalent.

That is where the 80.30 comes from for the measured. Then for the record 40.095 and 40.976 where do those come from, well we have to go back to the record data. Now these are heavy bearings here on both sides. I am going to have to take this bearing South  $88^{\circ} 25$  West, 40.11 chains.

Now I am going to have to compute the difference from cardinal, which is 1 degree 35 minutes. So I am going to take the co sine of 1 degree 35 minutes multiply it times 40.11 and that is going to give me 40.095 then, I am going to do the same on this other side. 87.40 northwest, that's what 2 degrees 20 minutes, so I am going to take the co sine of 2 degrees 20 minutes and I am going to multiply that by 41.01 and that is going to give me 40.976 and so that is where these numbers came from here.

All right we are doing the east-west now, so the computation is 80.30 in the measured is to sum of those two we just computed 40.095 and 40.976 as X is to 40.095 and I am going to solve for the westerly half mile and I am doing that just because it is going

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to make it real easy for me to compute the answer. So when we solve that we get 39.714. What are we going to do with that. Once again Remember that is a cardinal number.

I can go back to my measured data where I have my coordinates that are in the proper basis of bearings and I can take the 61.755 that is there and I can add to it the 39.714 which is the number I just computed for the easting difference (cardinal equivalent). What does that give me? It gives me 101.469 as you can see down here. That is the answer.

The way I have shown you to do this is proper and what it does for you is produce independently two single proportions but it produces the northing and the easting of the lost corner. The last thing you will do is inverse back to the controlling corners.

I will run through that again here in a minute or two and review for you just how that process works and make sure we understand that but let's make sure we recognize that bit maybe oversimplify in it if I say oh yeah double proportion that is just two single proportions. No. Double proportion you heard this the last hour or so it has to be on the correct basis of bearing true meridian. Then we have to use cardinal equivalents. The cardinal equivalents must be in the record and in the measured, so you have to adjust those now.

A lot of the GLO records don't need to be adjusted because they were already in cardinal or close enough. That's fine. This problem that we just worked the record was one of the lines was close to cardinal the other three were heavy bearings as we call them, so they require cardinal equivalent. It always does and so the measured you do that and that is really easy as you just saw because when you have coordinates because you can derive your cardinal equivalents without having to take the cosine of a bearing, or two bearings, adding it or whatever.

You can simply take your coordinate differences to get your cardinal equivalents in the measured. Then we take that information and apply it to the two single proportions now using cardinal values only and we come up with a cardinal value, which makes it really easy if you are using coordinates, which we all are anymore, you can add or subtract depending upon which side of

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the line you are going to solve for you can come up with that answer.

So you know I have heard people say well I have checked somebody's double proportion and they double proportioned and I missed them by two or three feet. Well hey it could be you. You could be wrong because you did not use cardinal equivalents or on the correct basis of bearings, so I would be very cautious of that and make sure that I am fully aware of how I did it versus how they did it.

Whether I want to accept or reject somebody else's corner because it can make a difference. I encourage you to experiment with it and see how much of a difference it makes.

I am going to review that double proportion process for a moment for you. Now you will understand what I was saying. The first step was to reduce the record distances to cardinal equivalents, then reduce the measured distances to cardinal equivalents, compute your proportions just like I did, north south separate from the east west obviously.

Remember that the answer you get out of those computations is cardinal. You are able to add or subtract depending on the proper direction that answer to one of your coordinates in your measured situation that allows you to establish the lost point you will come up with the northing and the easting coordinates of the lost corner and then you want to inverse from there from that corner out to your controlling corners because you don't know your bearings and distances to it.

What you have done is a double proportion on cardinal. So once you have determined that point, you are going to have to continue to inverse back out to get the final answers. To get your actual bearings and distances going into that section corner.

I encourage you to watch this again, if you have any trouble with that because it is a little different but the really important things are don't forget to reduce to cardinal equivalent, don't forget to be on your astronomic basis of bearing, your true bearing, and then apply it as we have shown you here and you'll do just fine.

### Double Proportion Process

1. Reduce record distances to CE
2. Reduce measured distances to CE
3. Compute proportions (N/S and E/W)
4. Remember: answer is a cardinal number
5. Establish lost point
6. Inverse bearings to controlling corners

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You might say well it doesn't make that much difference the record that I am using is all within 21 minutes, are cardinal, my measured stuff is pretty close too. Well you know I don't know where the cut off is that is kind of a dynamic relationship there. I would just say do it right every time and you never have to worry about well I wonder if I was close enough or I'll guess that I am close enough. Well don't worry that, just do it.

What did it take? I went pretty slow there, but you can compute these with a calculator in two or three minutes and or you could write a software routine that does it by the book because that is what we just did was by the book. And come up with a solution.

But the bottom line here is double proportions you know the example that I was going to give you bottom line is if you go out there on the ground, and let's say somebody else, you can't find any original evidence of the section corner and no one else could either and so they proportioned it, they double proportioned it well you tie them in and you miss them by two feet now you're going to decide am I going to accept them or not.

Well I will tell you what, before I make that decision I'd want to make sure that I did this on the proper method because if I am on the wrong basis of bearings that is why I am missing them two feet. And they did it properly and they were on the correct of basis of bearing. So do you see what I am saying, to evaluate to properly and legally, and if I can even say fairly evaluate other people's positions, you really need to be careful of what your position is and how it was determined before you compare it to theirs and say well they're two feet off. Well the reason they're two feet off, or five feet off or twelve feet off.

The reason they're off could very likely be because they were paying close attention to the rules and you weren't or vice versa. So let's pay close attention to those kinds of things.



**EXERCISE** Before moving on to the next topic, complete the “Double Proportion Exercise” which can be found in the Exercise section at the end of this study guide.

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We are still under double proportions but these point control methods again are for this situation, you use them where all lines run by the GLO in other words you know section corners normally have four lines coming to it.

But may one or two of those weren't run by it would have been double proportioned if they had run all the lines or if possibly you have identified a blunder in one of the lines then you are going to adapt the double proportion to a different method. These are all going to require cardinal moves so once again your basis of bearings is important and it is mandatory that you pay attention to it, in other words true bearing.

And all three of the methods we are going to look at require the running of the record measurement, so a record measurement, now what does that mean? We need to think about that because that is called, it allows you to run, it tells you to run record but the Manual has some interesting things to say about an index and what they mean by the record. So let's ask this question first before we go into these.

### Point Control Methods

(Double Proportions)

- Used where all lines were not run by the GLO, but would have been double proportioned if they had or you have identified a blunder
- Require cardinal moves; therefore our basis of bearings is important.
- All three methods require the running of a "record" measurement, which includes the application of an index.

So, what is an "index"?



**HANDOUT** Take a few minutes to read Jerry Wahl's *The Double Proportioning Made Complex* paper which can be found in the Handouts section at the end of this study guide.

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### Indexing

Let's ask this question what is an index and how does this work and how can I apply that? Indexing record is what they actually did and not just what the plat or notes say.

Now that is an interesting statement because we are all used to the record being what did the plat say and what did the notes say and that is it. But let me read you something because the Manual is interesting here. It talks about this in two different places in Chapter 7-14, 7-15 and Chapters 7-56 thru 7-58.

Let me read you the one from 7-14, this is the one, two, three, fourth paragraph in 7-14. An index correction for average error in measurement, which means it could be bearings and distances, and/or distances, if applicable should be made in applying these two rules and then they refer to section 7-56.

But then a real interesting sentence here, what is intended by record distance is the measure established in the original survey. Now that is a short sentence, but what that said is if they actually went a different distance than what the plat says and you are able to proof that, then may you ought to do that instead.

Now this is because indexing is a situation where you are going to adjust our record bearing that you don't have our measured, or excuse me record bearing or distance, that you don't have a measured bearing or distance directly on. How does that work? What is that?

Now let's understand. I'll show you here. Indexing 7-14, 7-15 and 7-56 thru 7-58 it can be applied to bearing and/or distance really, but it requires a substantial retracement that reveals a consistent, and I'm going, it is in italics, you may want to underline it, consistent pattern of error in the record measurements. Now here is what we've got.

Let's take a look at this slide. I am doing this a little out of order here so it threw everybody for a loop.

Take a look at this plat. Here is a township, I think in Oregon, where these sections out here were never surveyed. They are

### Indexing

- “**Record**” is what they actually *did*, not just what the plat or notes say
- BLM 7-14, 7-15, and Chapters 7-56 thru 7-58 discuss indexing
- Can be applied to bearing and distance
- Requires substantial retracement that reveals a *consistent* pattern of “error” in the record measurements

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unsurveyed. And so what we have here is these sections are 16 here and so they came up and they set their section corner here and they ran over here and so we have returns on both of these lines but there are no returns on the other two lines, so what we have here is a situation which would be a two-point control problem which we'll look at here in a minute.

But I am showing you where indexing comes into play. You see for me to compare, you know to run this line up my record is simply 80 chains and if that is all I have and I don't have any indexing available than I am going run whatever their record bearing and distance was. The distance probably 80 chains and whatever this is and we will talk about how that works. But the point is what if I had retraced a whole bunch of these lines in my project, and I had found that every time he says he went 80 chains, he was always a little short, maybe 79.90.

That, if you find a consistent pattern of that. That is what we call an index and an index then what I would do was if I was going to then run this line record distance, I would adjust it to 79.90 if that is what I had found was the consistent ratio on these other lines.

So that is an index and you can do it with bearing or you can do it with distance, or both. Now let's just remember something from when you took a statistics class, You know, you want to make sure your sample, using statistic's language, your sample is large enough, is a big enough representation of the population.

Now because when you apply an index, what you are saying is hey his chain was a little bit long or a little bit short. My example was long. His chain was a little bit longer than reality. And so when he says he just went 80 and I don't have anything to proportion against, but I know he went 79.90 on these others, I'll go 79.90. That's an index. But how many lines do you have to retrace to find that?

Now actually I think it was in the 47 Manual or maybe the 1930 or both maybe. It actually gave you some guidance on that and it talked about several miles. I think that is good advice. Now again, you may be finding quarter corners here and so you might have ten or twelve 40-chain increments where he said he went 40 and he only went 39.95 for example. Well great.



**DIAGRAM** A full size version can be found in the Diagram section at the end of this study guide.



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But what I am going to say is that when you take a **statistical sample** of something, it needs to be more than one or two, okay. Because you are saying this sample is representative of everything he did. It is just like these samples that they take for elections.

You know when they interview 1,000 voting adults or whatever you know and they come back and say, well they're for this initiative because 1,000 of them out of that 75% said; so what they are saying that 1,000 is a big enough sample to represent the rest of the population in your state or the voting district.

So what I am saying here is that you have to be very careful when indexing because you may have run two lines and they are both short and oh well then, he is running short all the time. Well, no. I think that you need to have a bigger sample. All I have to do is find a couple of lines where he is long and I have disproven your index completely. So think about that on that **sample size**.

Also let's understand that you don't just go take let's say you measure ten miles of this just as a you know, you must have a nice big project. You measure ten miles of this and it is all over the place. This one is long and this one is short this one is right on, this one is right on, next one is short, next one is long, next one is long. So you know you know you have a hodge podge of long, short and right on the record. You don't take those ten miles and **mean** that or **average** that. That is not an index.

An index is a **consistent pattern of error** in the record measurements. Now I am just going to tell you that you know there is not a lot of situations that come up where you are going to index it one of these one point, two point or three point control problems but in my career of 35 years I have never been able even when I have had opportunity to do an index, I have never been able to because I can not find that consistency.

Now I am not telling you that I am the standard by which to measure that I am just letting you know that I have known about indexing a long time and I have looked for it and opportunities for it and I have never found it because I am always all over the map.

Now if you've got ten miles and one of those miles are short and

## RESTORATION OF LOST CORNERS – PART 3

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the others are all long. Throw the short one out and average those other ones. Because you can have one odd-ball in there and again just go back to some of your basis statistics training and think about how you would sample things and how you might throw out the red herring or whatever. But let's recognize that indexing doesn't occur as often now GLO and BLM, they have had over the history you know of the Public Land System they have had tremendous opportunity to index. Why? Because up until just a few years ago, our projects were township at a time.

Well you know when you do the entire township, you've retraced every mile of line in that township, you've got a really good picture of what that surveyor did and how long his or short his measurements were or his angles were always to the right or always to the left. You know his bearings I should say. So you could figure that out. But again indexing is there and I wanted to explain it to you because all three of these point control methods mention it. So I showed you that example a few minutes ago. Let's talk about a **two point control**.

Two point control is used where the corners, that normally have been double proportioned, have only been established from two directions.

What we do is run record bearing and distance from each of the corners that we do have and we set temporary points. We are going to have two separate temporary points there. You make a cardinal move from each of those true points which means you are going to inverse back to your controlling corners to get your bearings and distances.

What you are going to see is that two point control is not a bearing bearing intersection and it is not a distance distance intersection either.

So once again here is the problem we have a section corner which has been established by the GLO or BLM and the problem is that these lines were never, we have no returns on those.

So we can't do a normal double proportion because a normal double proportion requires us to go, I'm assuming the quarter

### Two Point Control

- Used where corner only established from two directions
- Run record B&D from each to set temp
- Make a cardinal move from each to the true point
- Inverse back to control
- This is not a B-B or D-D intersection!

## RESTORATION OF LOST CORNERS – PART 3

corners are lost here, requires us to go between controlling corners each way, but we don't have that option here so you have got to come up with a way to set this lost corner, and the way the Manual says, and you can read about that in 7-14 and in fact it is the paragraph right above the index correction discussion.

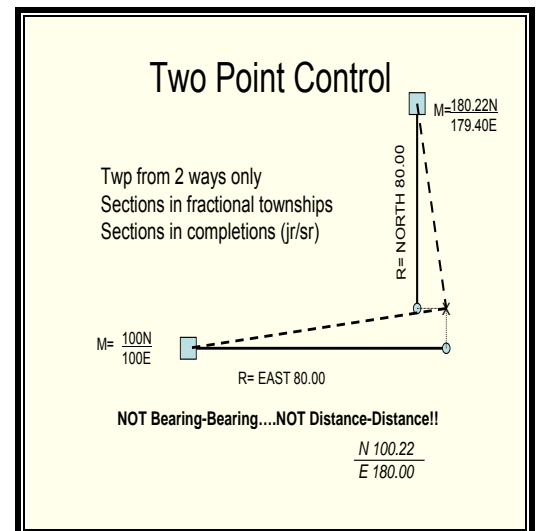
It says where the intersecting lines have been established in only two of the directions, the record distances to the nearest identified corners on these two lines will control the position of the temporary points. So there is your record information. Then from the latter, the temporary points, the cardinal offsets will be made to fix the corner point.

What they are telling, going back to the slide, what they are telling us is that I've got to say that this is my found corner down here and let's put a quarter corner in let's not be so unfair to the quarter corners, and so I have to run record bearing and distance from here and I've got to run record bearing and distance from here and they are probably going to come out in two different places and if they don't it's an absolute miracle. So let's look at one of these and see what it is we are going to do.

**Two point control** is what we have here. And here is your examples of when you use it and that sort of thing. You can have this in completion surveys where it looks like the whole town was done at one time now but it was done in two or three different surveys you remember that from course 2, talking about completions.

So you want to think about that. How you restore things. Now so here is our situation, I've got a section corner that is lost. It is over in here somewhere and I don't have any quarter corners here so I have a found section corner here and a found section corner here, now the record, I kept it cardinal to keep it simple, the record on this line was east 80 chains, the record on this one was north 80 chains.

I've also given you coordinates in chains that are from your survey. That could be a traverse that could be GPS that could be whatever. You determine coordinates on those positions. So if this corner here is at 100 and 100 and go due east 80 chains, that is going to put me at, because I am going due east it will still be



## RESTORATION OF LOST CORNERS – PART 3

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down here my temporary is going to be at north 100 and at east 180. Right? North 100 and east 180.

Cause I went due east 80 chains exactly from this coordinate. So that gives me a temporary point down here. Then I am going to go up to this other found existent section corner and I am going to run record bearing and distance from it.

As you can see its coordinates, northing coordinates that you found it to be is at 180.22 north, 179.40 east. So if I go due south 80 chains from that coordinate, I am going to set a temporary point and that temporary point here is going to be at a coordinate of 100.22 north because I went due south 80 chains and 179.40 east. But, so what we have done is exactly what the Manual said, we have run record bearing distance from the two corners, set temporary points and then what did the Manual say to do? It said to move cardinal offsets, cardinal offsets, well you can do that in math, see.

Because if you are going due north from this temporary point then it is going to maintain the same eastern coordinate as your temp had. So the final section corner that you are setting here is going to have an easting of 180 even. Whereas I am going to do the same from here, this one the temporary point right here, has a northing of 100.22 and an easting of 179.40 well I am going to make a cardinal offset. I am going to go due east from this to where these intersect. See, cardinal offset to where they intersect. Well if I go due east, the northing will remain the same, the easting will change, but I am not going to use the easting for that. I am going to use the northing. Therefore, there is our answer.

So you see what I am saying here is you run the record bearing and distance, you set your temps, you do this all in math, and then you make cardinal offsets, which means you maintain either the easting coordinate here or the northing coordinate here so that it was a cardinal move and that gives you the northing and easting of the corner that was lost that you are resetting. And now if you remember the last step that I had in there was now you will have to inverse from there back down to the two controlling corners so that you actually get your bearings and distances there. And if you look closely at the drawing you will see that this is not a bearing-bearing intersection, because a bearing-bearing

## RESTORATION OF LOST CORNERS – PART 3

intersection would have been here right? If the two record bearings had met. And if it was a distance distance intersection, it would have been somewhere over here but not at the same point. But it is not. It is not a bearing bearing intersection or a distance distance intersection, it is a two point control and that is how that is done.



**EXERCISE** Before moving on to the next topic, complete the “Two-Point Control Exercise” which can be found in the Exercise section at the end of this study guide.

Let’s go on to **three point control** which is also well if fact, let me read that to you first here. That is the first paragraph in 7-13 where the line, this is still double proportions we are under and it is talking about section corners, things that would normally have been double proportioned where the line has not been established in one direction only from the missing township or section corner, the record distance will be used to the nearest identified corner in the opposite direction.

So what they are saying is that one, we have three of the lines coming in but one of them does not have something to proportion to. We have a line that we can proportion along to get our point but in the other direction it was just stubbed out that way or ended that way.

So let’s take a look at that then. Three point control what will that require? You are going to do a proportion on the through line or the complete line that runs through there and that is going to be with cardinal equivalents which unless there is a bearing break, it probably won’t matter. We run record bearing and distance on the single line, the line that does not have a continuation to it and you can index on that, if that is applicable or appropriate.

You make cardinal moves from those two points, those two temporary points, once again that sets the actual points of cardinal moves, your basis of bearings is absolutely critical okay and you inverse back to your controlling corners for your final courses.

### Three Point Control

- Do a proportion on the through line, using a cardinal equivalent
- Run record B&D on the single line (indexing?)
- Make cardinal moves from these two results to set actual point
- Inverse to controlling corners for final courses

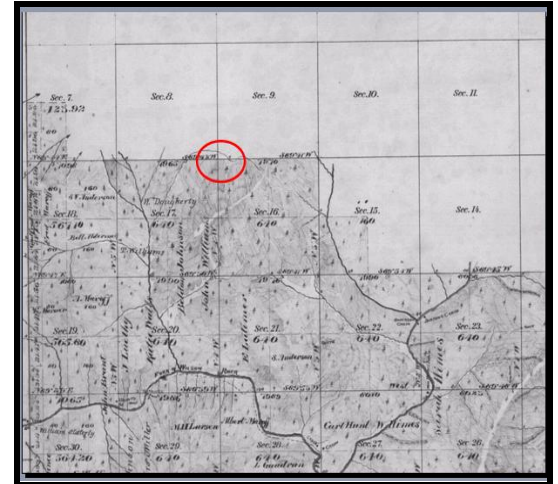
## RESTORATION OF LOST CORNERS – PART 3

Here is a plat. Same plat, just different part of it now. We just did this corner a few minutes ago, two point control. But notice on this section corner, we have three lines coming into it. We have this one, this one and this one. Now see, I can do, let me make us some corners here, let's say all the quarter corners are in here. So I am putting all those quarter corners in. I can do a normal east west proportion, double proportion with cardinal equivalents here. I can do that here.

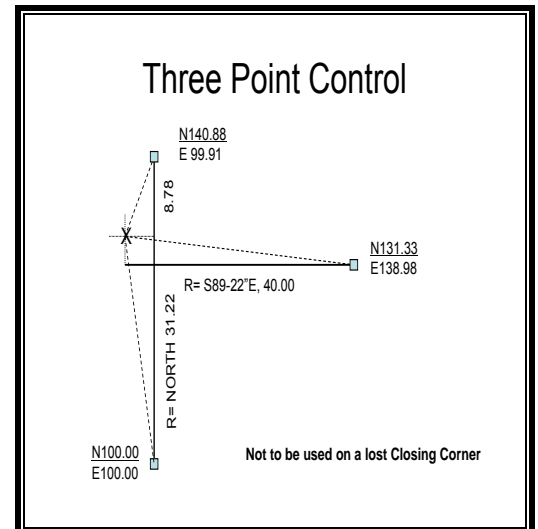
But going north and south, I don't have anything to proportion to out here. So I am going to have to run record bearing and distance from this quarter corner up to here and I can **index** that but you can see what is happening when you do the proportion east west, it is going to give you an easting, when you run record bearing and distance up here it is going to give you a northing that is going to be those cardinal offsets just like we saw on the two pointer except this one has three point control. So let's see how this works. In the drawing that I have given you, you can see the record dimensions, I kept everything, but not everything cardinal, but pretty close.

We've got north 31.22 chains there and 8.78 still north there is where this section corner or township corner hadn't been set and then there was a line only run off to the east here and there is our bearing 89.22 east 40 chains.

Now the, so what we just read and what we just talked about as we are going a normal proportion because that is a complete line, the through line. It went all the way through. I am going to do a normal proportion there but on this one I am going to run record bearing and distance. So the normal proportion north south is going to give me a northing, the run record bearing indexed if possible is going to give me an easting, and so we are going to make these cardinal moves to come up with a point. So how does this work?



**DIAGRAM** A full size version can be found in the Diagram section at the end of this study guide.



## RESTORATION OF LOST CORNERS – PART 3

On the north south I am going to do just my normal computation like we did before when we take cardinal equivalents. The record is 40 chains, 31.22 plus 8.78 is 40. It is north here so there is no cardinal reduction. They are cardinal equivalents but they are close enough so that you don't have to worry about it. And then I am going to compare that to my measured cardinal equivalent.

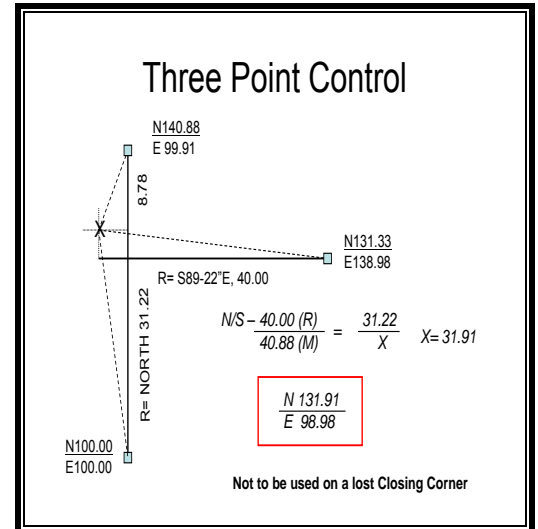
Well I can do that quite easily, right? Because I can take this northing coordinate and this northing coordinate, these are my measured coordinates in chains to the corners, existent corners that I found, the controlling corners, and so I know that I am at 40.88 because that is my difference in northings. So I've got 40 in the record is to 40.88 in the measured as 31.22 that was this one here, is to "X", "X" equals 31.91.

So just like we did on double proportions, I can take the 31.91 and simple add it to that northing, because I solved this line. So that means that the temp, if you will, that this is going to be at is going to be at 131.91, of course that is going to be our northing answer because we are going to make a cardinal move off of that.

So it will maintain its northing. So 131.91 is going to be the northing of this lost corner. And then how do I do the east west? Well, we have a coordinate over here. At this point and we are going to run record bearing and distance there. Now record bearing and distance is 40 chains but if you go 89.22 see we are going to reduce this to cardinal, if you actually ran 89.22 at 40 chains it would, the cardinal equivalent would be less than that.

You would not be 40 chains due east or west, of this point, you would be in the cardinal. You would be something slightly different. And see that is the whole idea, if you run record bearing and distance that is on an angle.

And so your distance, your point here, the easting of it is not going to be 40 chains from here. So just think about it and it will make sense. So what I want to do is go in this case south 89.22, well I'm going to go north west, sorry, north 89.22 west, right, because we are going back the other way. North 89.22 west, so I have to take the co sine of 38 minutes, right, times 40 chains and I believe that comes out to real close to 40 chains.



## RESTORATION OF LOST CORNERS – PART 3

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Now you want to always check this, don't assume. But it looks like at least the answer I got that must be right, it comes out right at 40. Because all I got to do is take this easting coordinate and subtract my distance here, not the 40, but the cardinal equivalent, which is probably 40 anyway, and it looks like it is and that gives me 98.98 and that is of course the easting of the corner.

Now a couple of thoughts here while we are looking at this, in the record we had a straight line here and another line that went out almost at 90 degrees. Understand that I have exaggerated here just so that you can see it but you could very easily end up with a bearing break out here, and when you are done, this thing could actually have an angle break in there and you might.

Many a surveyor says well I can't do that because the record says this is a straight line. Well what you are doing if you are going to insist that that be a straight line, then you are throwing out this line.

You are saying that that line doesn't matter or doesn't count. And I want to remind you that in the first hour, the first video lecture, we read some things in the Manual that said that you **weigh equally all parts of the line**, that you give equal relative weight to all parts of the line, that you don't prefer two corners over the third or any more you weigh all parts of how that corner was established, so if you see on that slide again, the problem is if you want to force this to be straight, and I don't think that that is a good idea.

Because what you are saying is that only the positions of this corner and this corner matter and this corner doesn't matter and this measurement doesn't matter. Yes it does. Under the law and under the way the Manual describes three point control, it does so. It is possible, it is very rare that you are going to have it this heavy of a bearing break, although the more this line is offset from the center of this line, the more likely you are going to have a heavier bearing break at this point.

But it is very likely that it is going to end up with something that doesn't perfectly resemble that but that is because you weighed all parts of the line, you took all three corners and you used the data



## RESTORATION OF LOST CORNERS – PART 3

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between them to come up with this position, using the book and that is where we came up with that coordinate.

That is one thing that I wanted to mention. One other here, three point control, it doesn't happen a lot. And I want to make sure you realize that and notice this statement down below here, This is not to be used on lost closing corners. There is an awful lot of places where it might look like a three point control problem, but in fact the corner that is missing is a closing corner. We're going to talk about that a little later in this course for lost closing corners. But understand that the three point control only really applies when the corner that you are trying to reestablish has returns coming into it that are all from the same survey.

Well, that may be after, if you are retracing a resurvey then what was a closing corner has now been adjusted to the line barring any major problem there, you might be able to use three point on it then, but really three point is not designed for closing corners. That's the bottom line. So keep that in mind.



**EXERCISE** Before moving on to the next topic, complete the "Three-Point Control Exercise" which can be found in the Exercise section at the end of this study guide.

Now we are going through these point control methods and there is one more here and that is **single point control**. It's a pretty easy one. It is over in 7-56 of the Manual. Let me just read that real quick to you. It is kind of you know not the best worded but it is under 7-56.

They call it **original control**. Original control, where a line has been terminated with measurement in one direction only. That means they ran out and stopped measurement. There is no connection anywhere just that one that you made in one direction only.

A lost corner will be restored by record bearing and distance, counting from the nearest regular corner, the latter having been duly identified or restored. And then they talk about indexing and indexing here as you can see can also apply. Now single point control.

## RESTORATION OF LOST CORNERS – PART 3

Let's take a look then this is going to apply where the line was established, the corner was established in one direction only.

Those can sometimes be meander corners where the lake was too wide or the river was too wide or at the ocean, quarter corners that they stubbed in, there are some other situations, I'll show you apply here are some others. 7-56 we just read it. It told us to run record bearing and distance. It did say that indexing is allowed, if you can prove it. And as I have said to you, maybe you can't.

Take a look at this plat. Here is a big canyon, impassable. All right, this is up near Sedona, Arizona. And these big canyons have the red rocks down in here.

But when the GLO did the stuff up on the top up here, this is up on a mesa with pine trees and that sort of thing. Notice that they did not complete the sections out into the impassable canyon. Now here this is an interesting one, they started at this section corner, came over here and set a quarter corner, came over here and set a section corner, came over here and set a section corner and then stopped.

And there are no measurements coming into this section corner or this quarter corner, you know going on out that way or something. There is nothing there. So the question here is, how would I reset any of these corners, frankly? You know, if all three of these were lost and this is found, well then you would run record bearing and distance, record bearing and distance, record bearing and distance.

That is what 7-56 is telling us to do. That is the kind of situation we have. Well as you can see on this plat well there are a number of single point control problems on this one as well, okay, and there is another one down there. Here is a two point but that is not our subject. But you see you get combinations of it in these surveys like this. So that is what we are talking about, a single point control and how to set it and again as the drawing showed us, well just run whatever the record is, you can index it, run record bearing and distance.

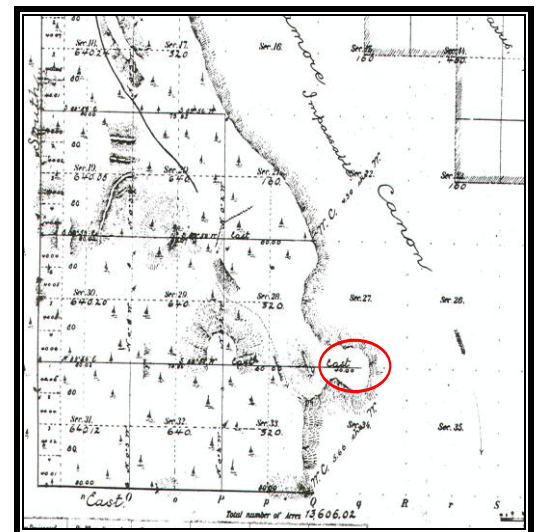
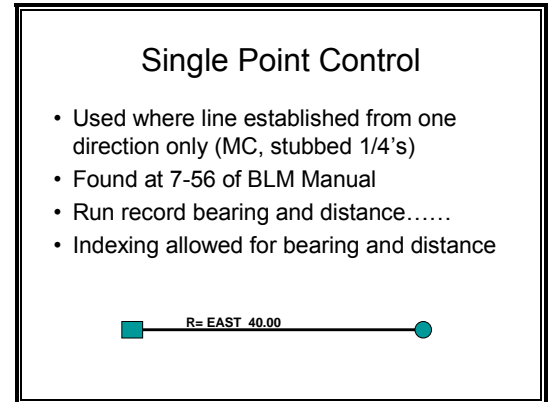


DIAGRAM A full size version can be found in the Diagram section at the end of this study guide.

## RESTORATION OF LOST CORNERS – PART 3

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Now frankly, with any of these point control methods, but especially the single point control. If I ran here due east 40 chains and there is a fence that is slightly off you know and the fence ends at 40 chains you know I would at least be looking at that as a possibility of evidence because you know indexing with these single point control, who knows what is goes on. Each line is kind of unique. But what you want to be careful of is you don't want to just say well okay here's like they stubbed south okay, you don't want to say well okay I'll look at this half a mile here and they said they went 90 degrees so I'm just going to turn 90.

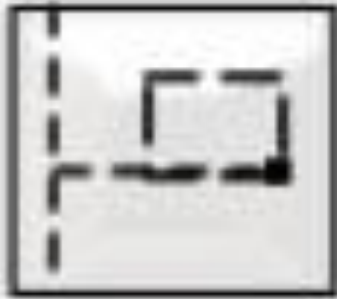
Well you know you want to be careful of that and say that that is indexed. That's not. Because you see I could just as easily go and take the half mile at the other side of your corner and turn 90 there if that's the record. And I will miss this. I will miss your point. So you want to be careful with those things. But I do look a little bit more in these single point control situations. Look a little bit more at what's going on on the ground and it may, if you will, allow you to do a local index or just accept, or what I am saying is accept the fence corner as the best available evidence. But notice that I had a lot of "what ifs" in there.

Here's the bottom line, the Manual says run record bearing and distance so if you are running record bearing and distance to a single point control a corner than I am assuming you have not found any acceptable evidence. So in other words you have already looked at the fence and decided it wasn't that good. But that's single point control. That's how that works.

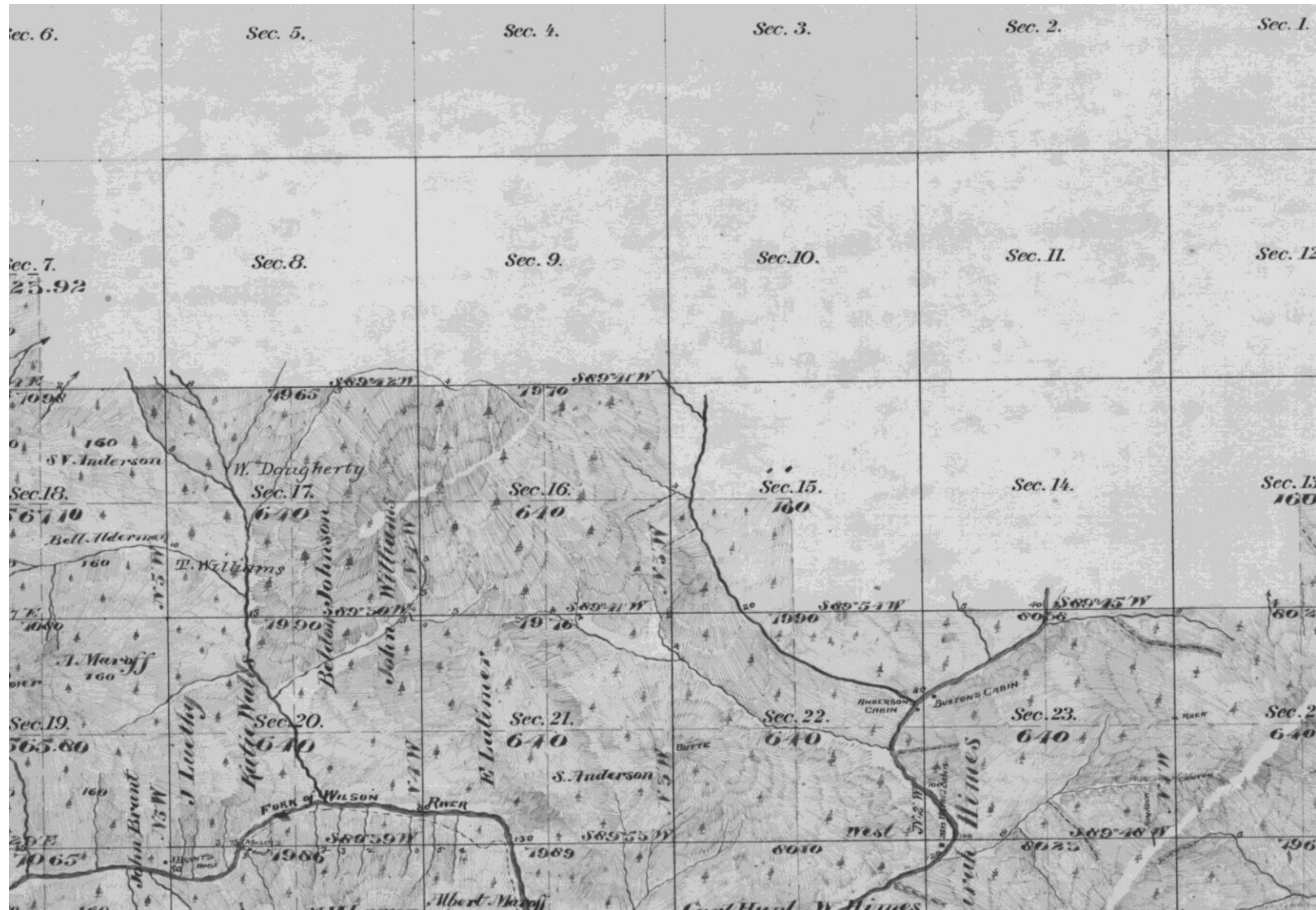
Now we are going to take a break here in this video lecture and come back with another one here in a few minutes and start talking about some things that well the grant boundary method especially which takes us away from the rectangular system and goes into the non-rectangular entities so we'll be talking about that on the next lecture. So Ron and I will both see you over there.

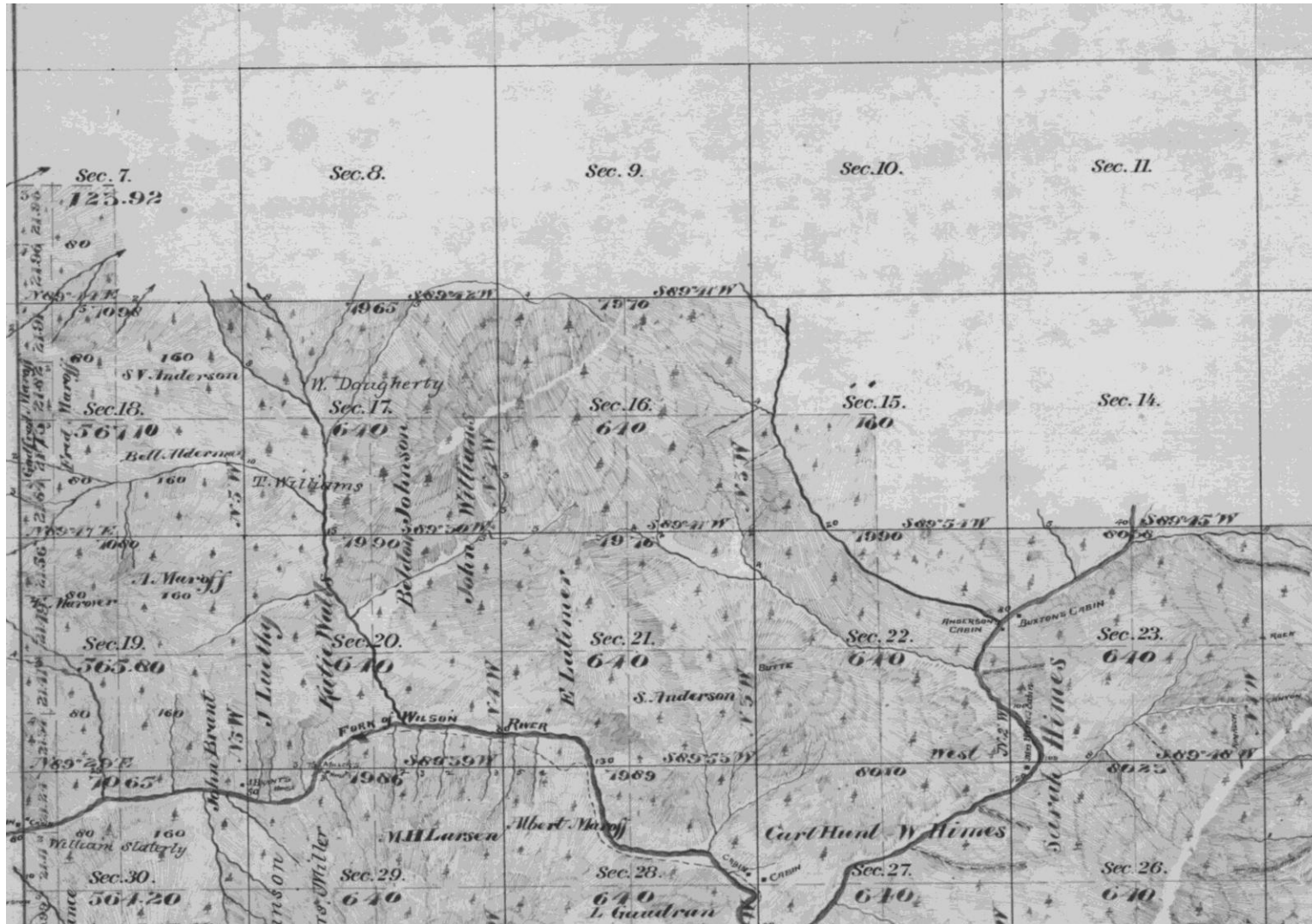


**EXERCISE** Before moving on to the next topic, complete the "Original Control (Single Point) Exercise" which can be found in the Exercise section at the end of this study guide.

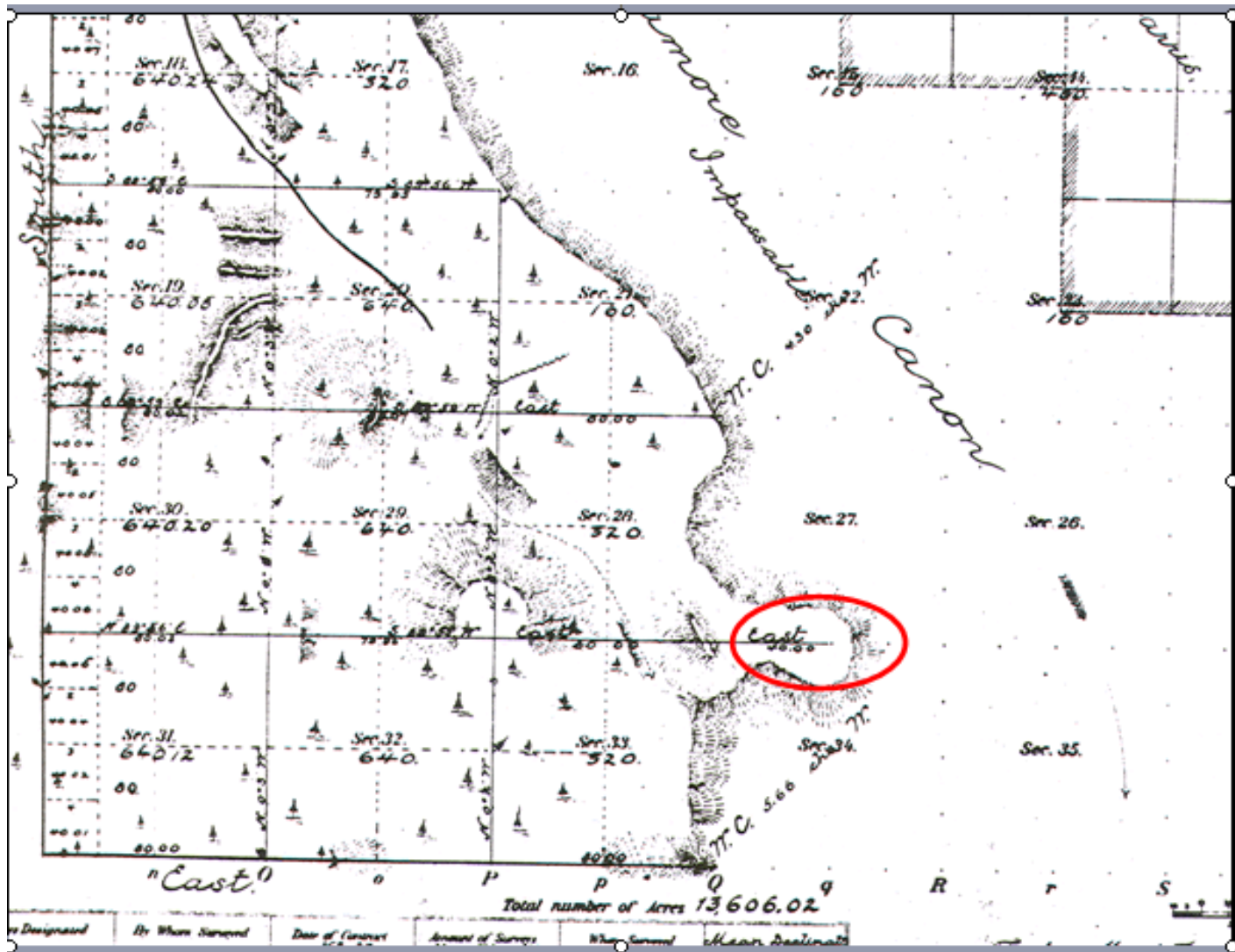


**DIAGRAM**











## **EXERCISE**





**N-S Proportion:**

Record latitude between controlling corners: N. 80.00 chs. = 5280.00 ft.

Retrace latitude between controlling corners: N. 5302.69 ft.

Record proportion is midpoint, therefore the latitude to the lost sec. cor. is: 2651.35 ft.

N. 10000.00 + 2651.35 = N. 12651.35

**E-W Proportion**

Record:

Cardinal equivalent of the departure of the E ½ mile:  $\sin 78^\circ 26' \times 36.06 = 35.328$  chs. = 2331.63 ft.

Cardinal equivalent of the departure of the W ½ mile:  $\sin 89^\circ 54' \times 40.22 = 40.220$  chs. = 2654.52 ft.

Record departure between controlling corners: 4986.17 ft.

Retrace departure between controlling corners: 5093.30 ft.

Retrace departure  $\div$  Record departure = K

$5093.30 \div 4986.15 = 1.021490$

K x Record departure of each segment of the line = Proportionate departure of each segment of the line

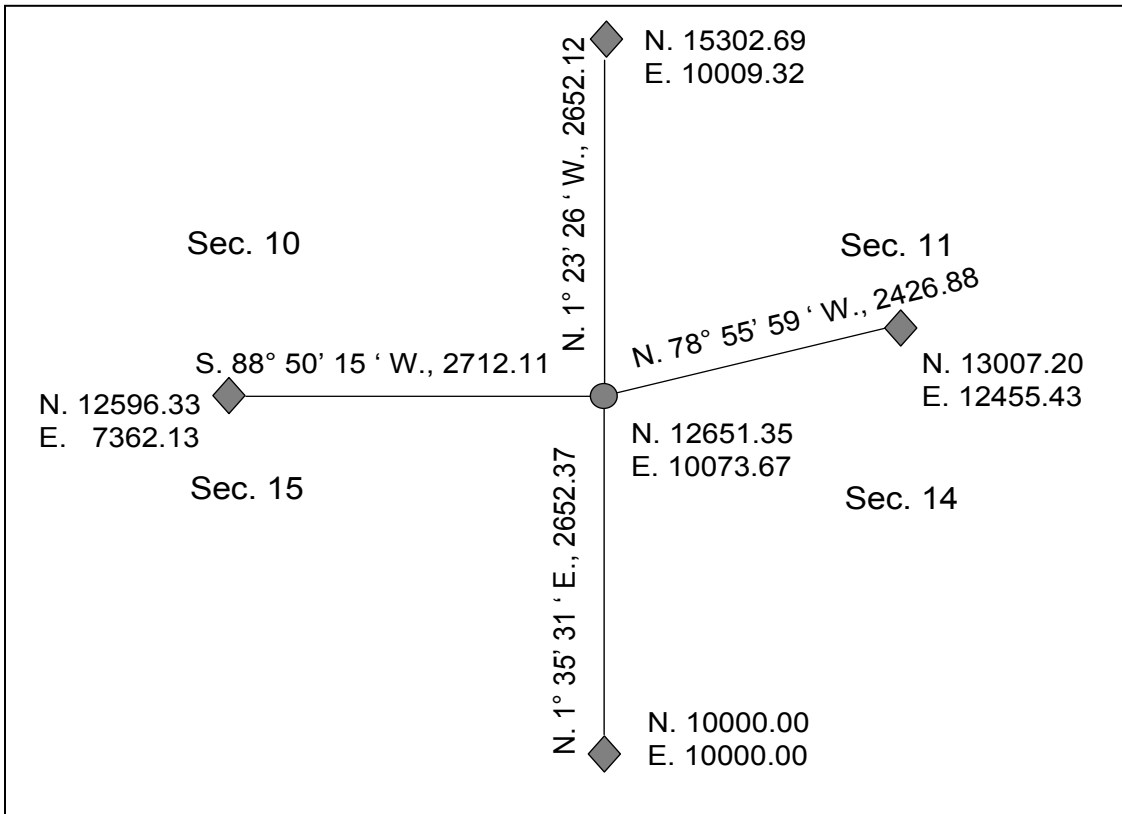
$1.021490 \times 2331.65 = 2381.76$  ft.

“  $\times 2654.52 = 2711.56$  ft.

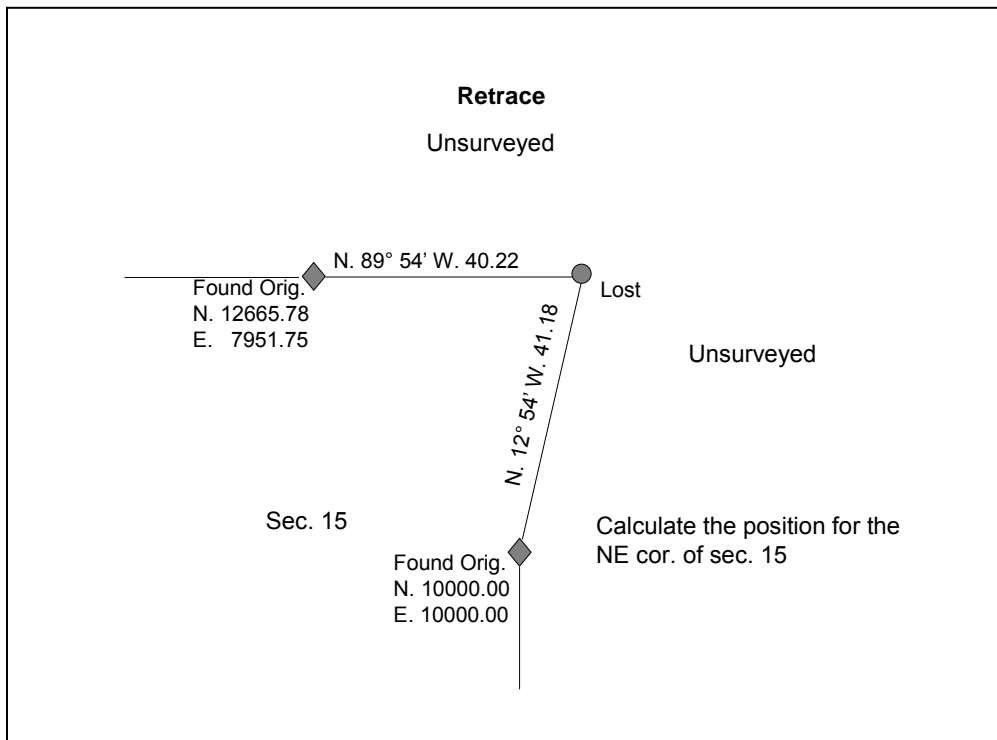
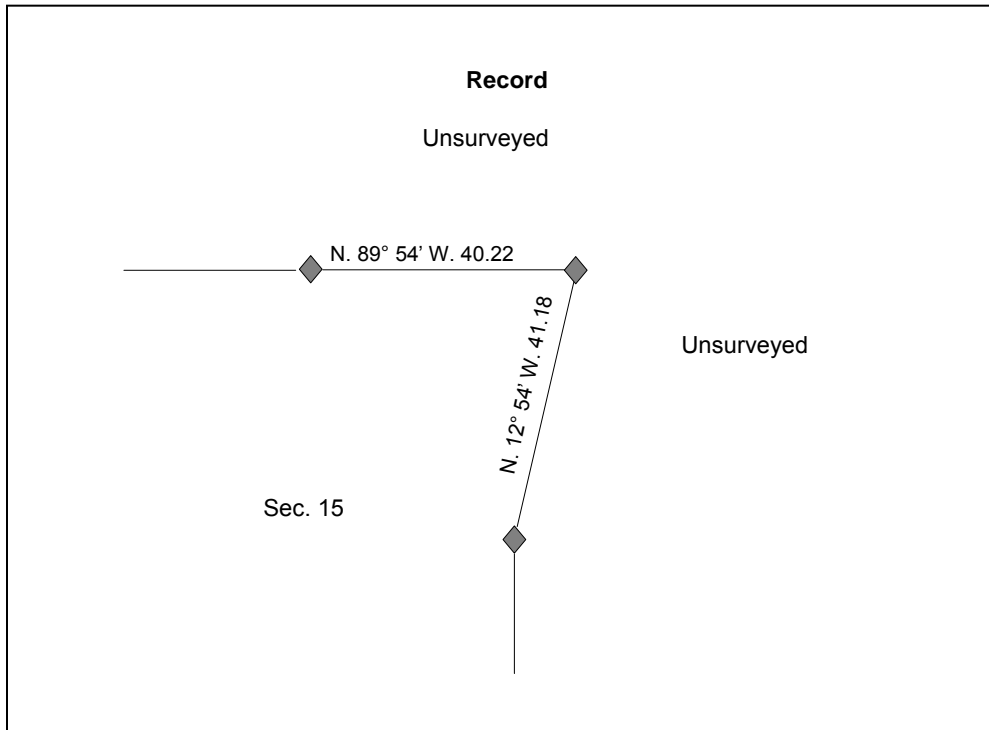
E. 12455.43 - 2381.76 = E. 10073.67 ft.

**Proportionate Position of the Section Corner**

**N. 12651.35  
E. 10073.67**



## Two Point



**Calculate N-S**

Record latitude:  $\text{Cos } 12^\circ 54' \times 2717.88 \text{ ft. (41.18 chs.)} = 2649.28 \text{ ft.}$

$\text{N.10000.00} + \text{Record Latitude} = \text{Proportionate latitude for the corner}$   
 $\text{N.10000.00} + 2649.28 = \mathbf{N.12649.28}$

**Calculate E-W**

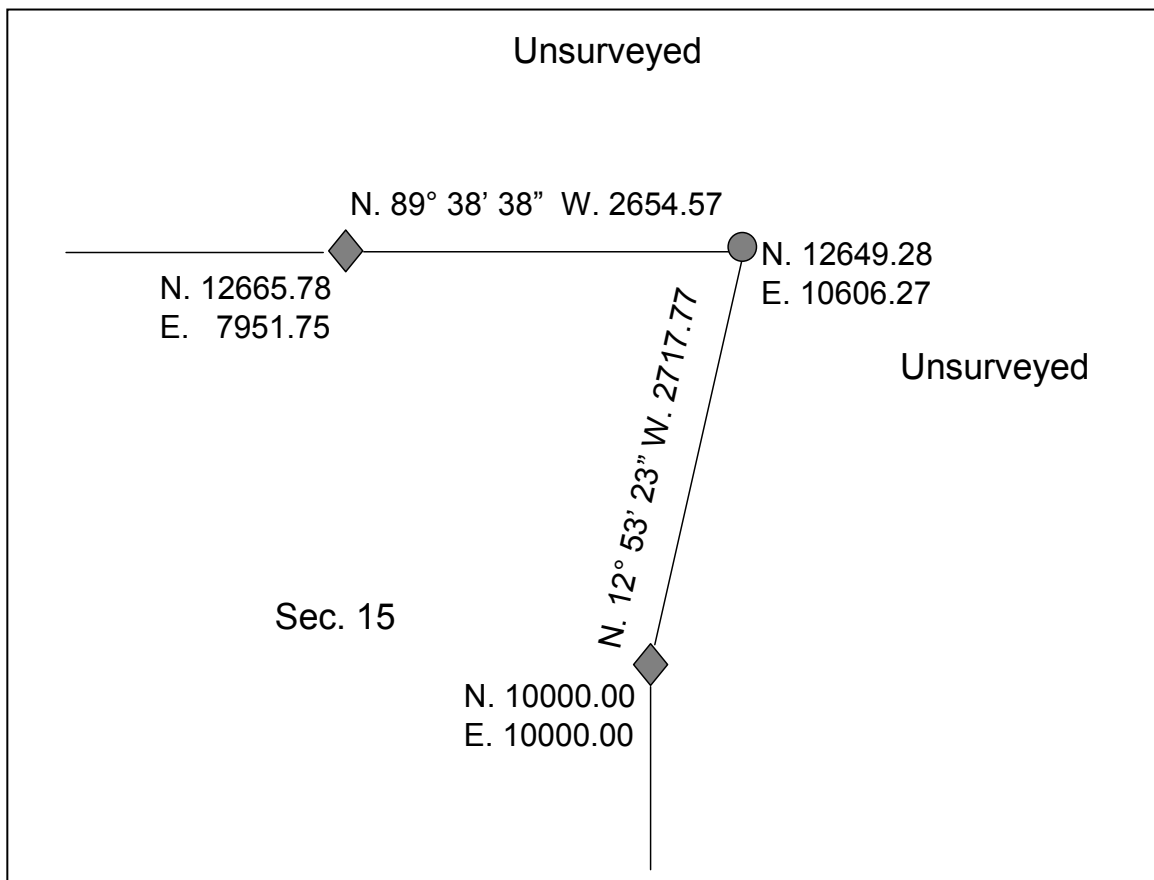
Record departure:  $\text{Sin } 89^\circ 54' \times 2654.52 \text{ ft. (40.22 chs.)} = 2654.52 \text{ ft.}$

$\text{E. } 7951.75 + 2654.52 = \mathbf{E.10606.27}$

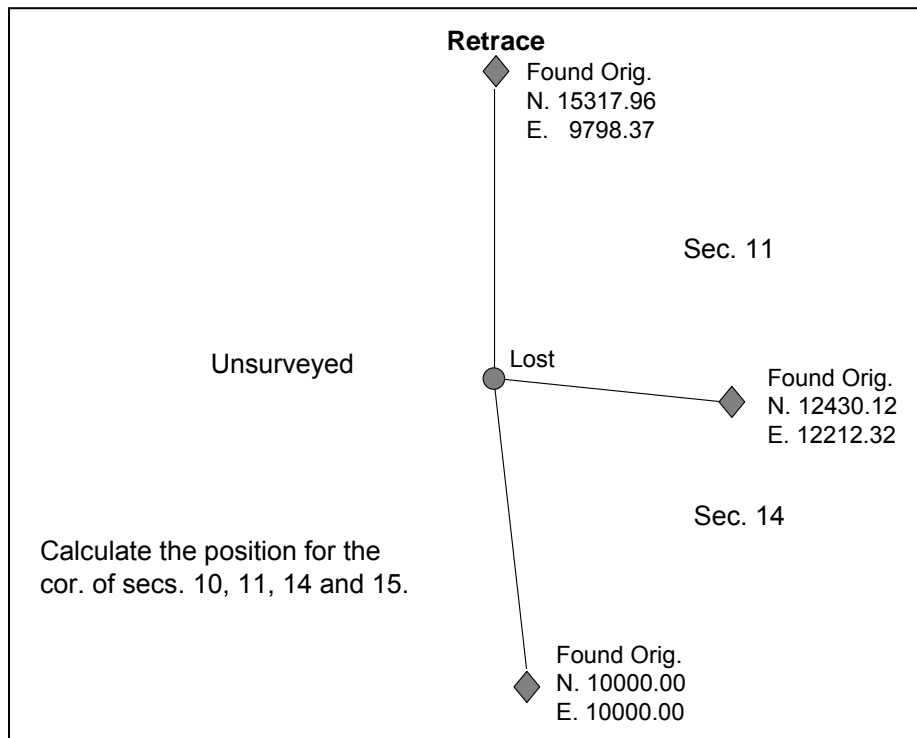
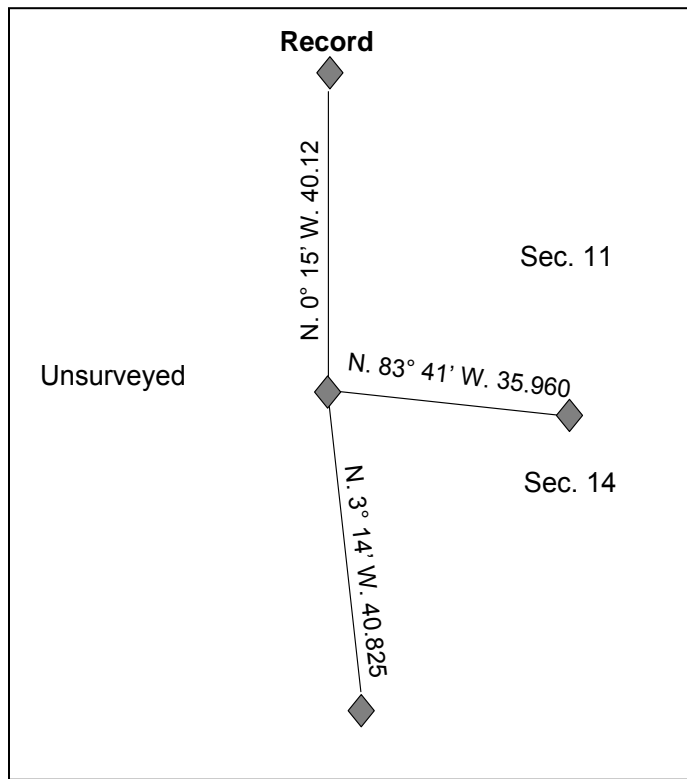
**Proportionate Position of the Section Corner**

**N.12649.28**

**E.10606.27**



### Three Point



**N-S Proportion:**

Record latitude between controlling corners: N. 5338.06 ft.

Retrace latitude between controlling corners: N. 5317.96 ft.

Retrace  $\div$  Record = K

$$5317.96 \div 5338.06 = 0.996235$$

K x Record latitude of each segment of the line = Proportionate latitude of the line segment

$$0.996235 \times 2690.16 \text{ ft. (40.760 chs.)} = 2680.03$$

$$\text{“} \times 2647.90 \text{ ft. (40.120 chs.)} = 2637.93$$

$$\text{N.}10000.00 + 2680.03 = \text{N.}12680.03$$

$$\text{N.}12680.03 + 2637.93 = \text{N.}15317.96$$

**E-W Proportion**

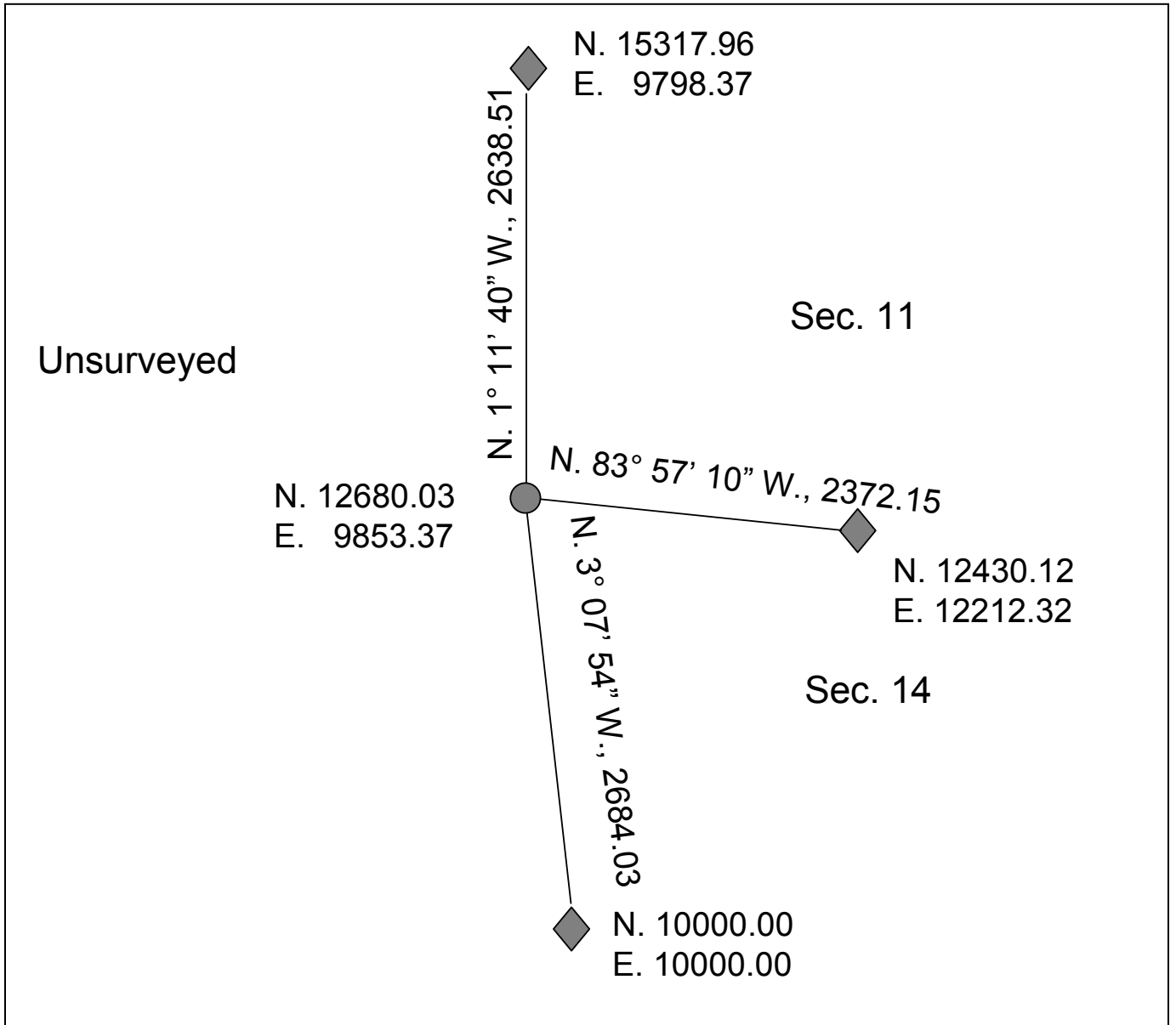
Sin  $83^\circ 41'$  x 2373.36 ft. (35.960 chs.) = Record departure (minus because the line is West)

$$0.110023 \times 2373.36 = -2358.95 \text{ ft.}$$

$$12212.32 - 2358.95 = \text{E. } 9853.37$$

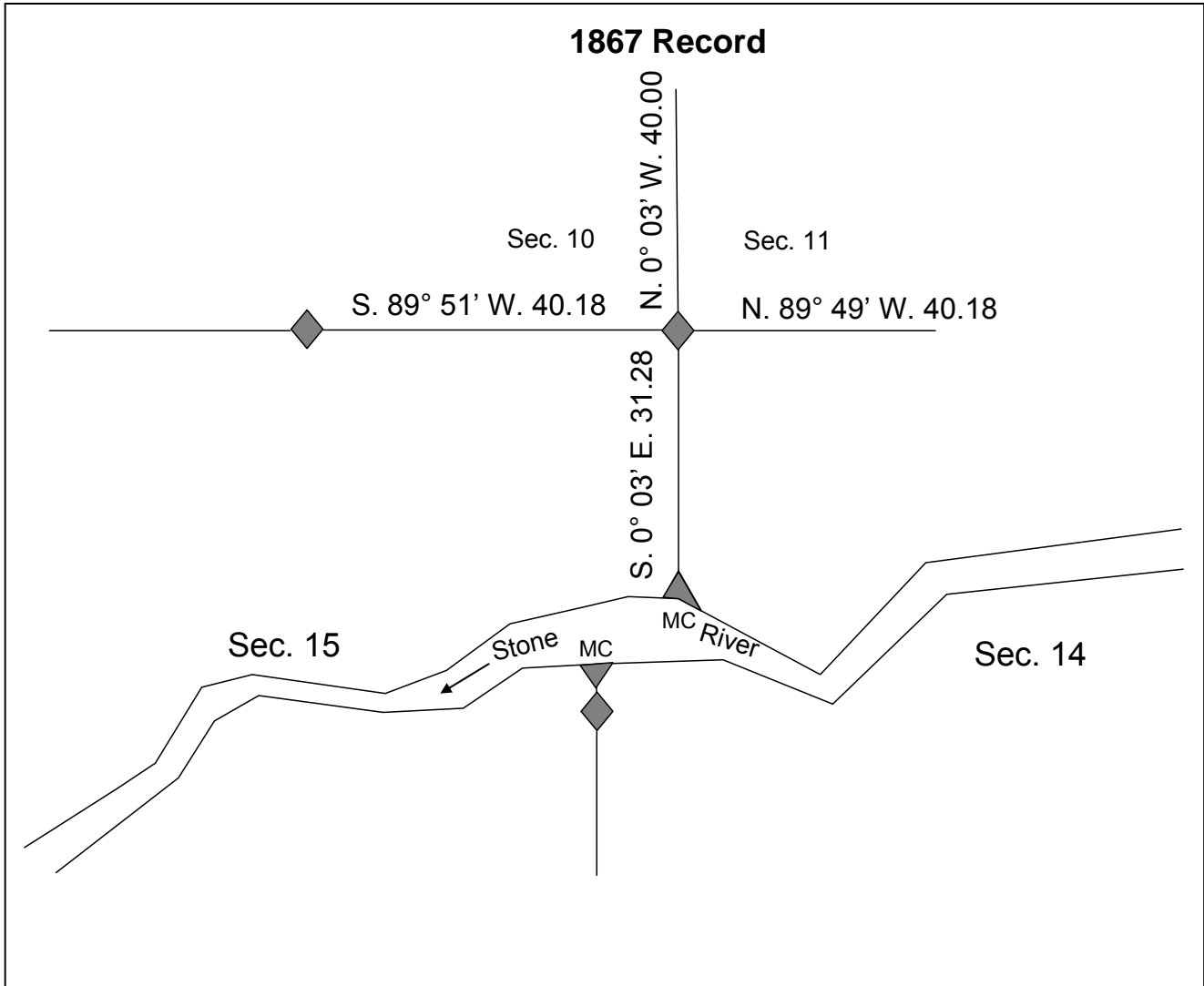
Proportionate Position of the Section Corner

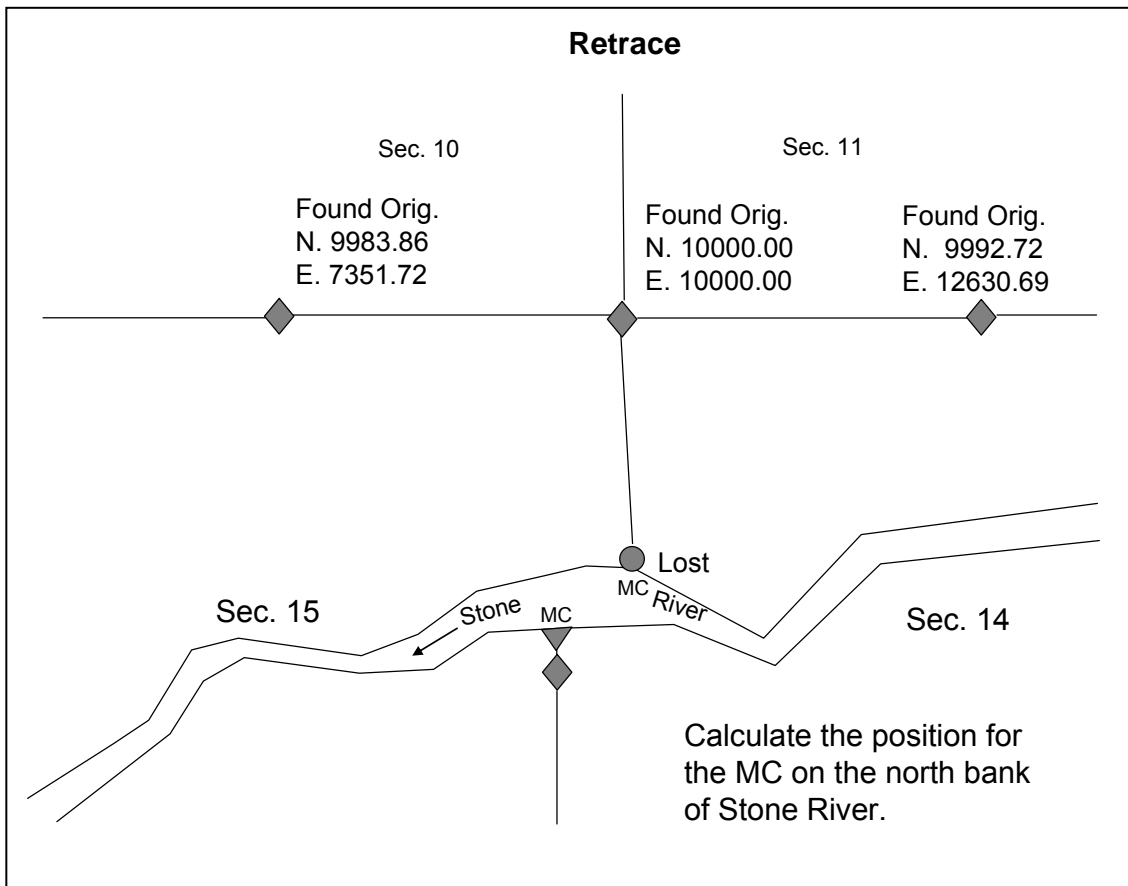
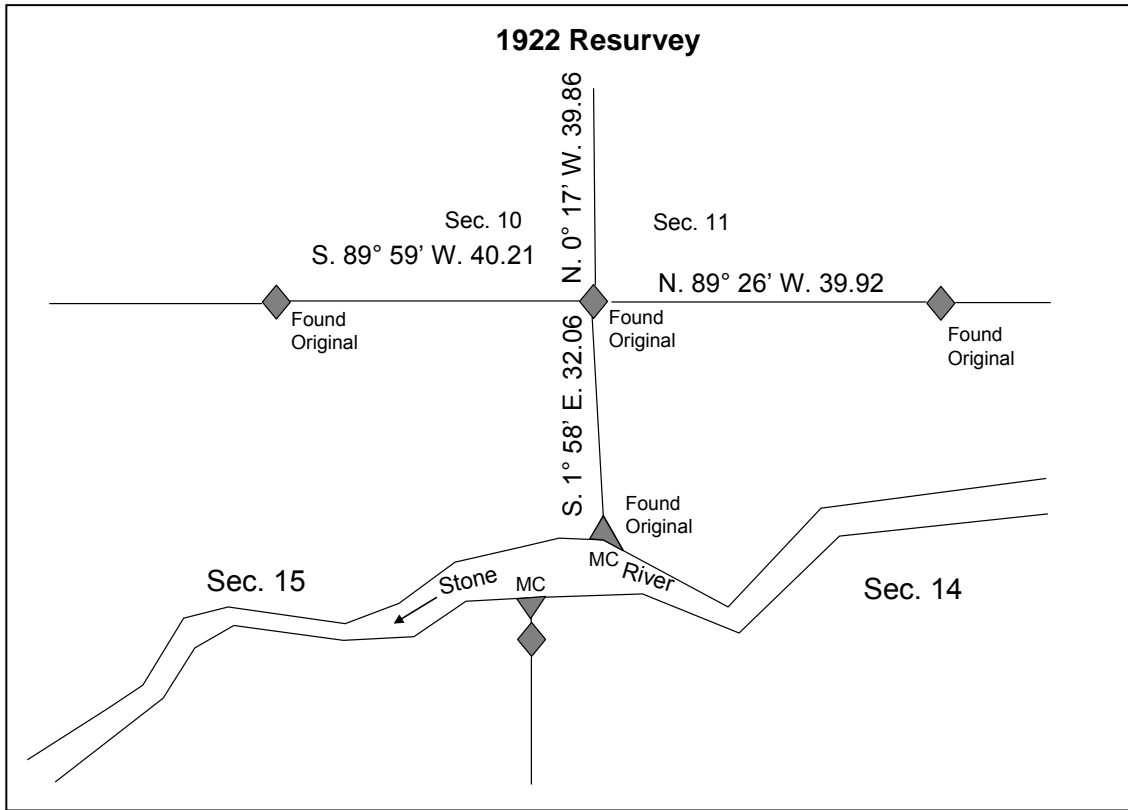
N. 12680.03  
E. 9853.37





**Original Control (one point)**





**Sec. 7-56 Original Control**

Where a line has been terminated with measurement in one direction only, a lost corner will be restored by record bearing and distance, counting from the nearest regular corner, the latter having been duly identified or restored.

An index correction for average error in original measurement should be used, if appropriate, as discussed in section 5-29.

**Calculate N-S**

*(An index correction is not appropriate because we have not retraced enough of the retracement survey)*

$$\cos 1^\circ 58' \times 2115.96 \text{ ft. (32.06 chs.)} = \text{S. } 2114.71$$

$$\text{N. } 10000.00 - 2114.71 = \text{N. } 7885.29$$

**Calculate E-W**

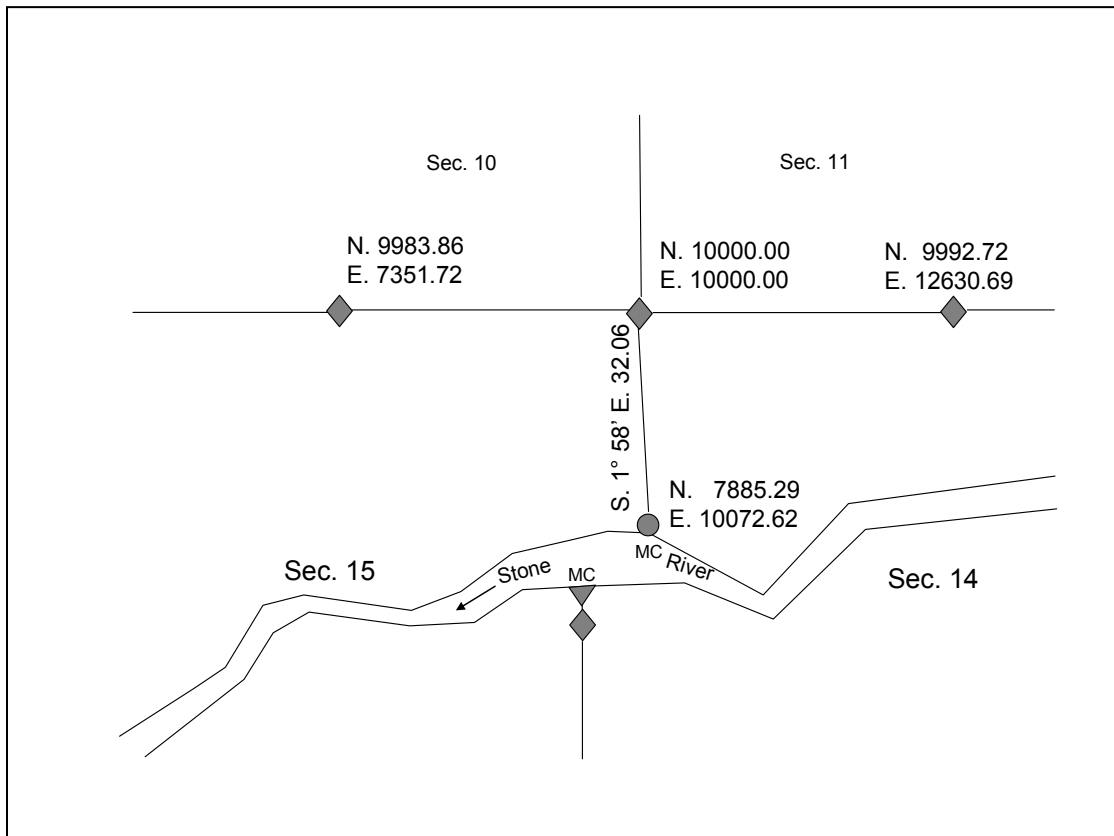
*(An index correction is not appropriate because we have not retraced enough of the retracement survey)*

$$\sin 1^\circ 58' \times 2115.96 \text{ ft. (32.06 chs.)} = \text{E. } 72.62 \text{ ft.}$$

$$\text{E. } 10000.00 + 72.62 = \text{E. } 10072.62$$

**Proportionate Position of the Section Corner**

**N.7885.29**  
**E.10072.62**





**HANDOUT**

# DOUBLE PROPORTION MADE COMPLEX

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## ABSTRACT

Our ever increasing ability to accurately measure make it more critical that we understand the geodetic and legal concepts behind some common survey principals. As presented here, the process of determining the position of a lost section corner under the Public Land Survey rules is well known, and is even generally a matter of actual statute law. On the technical side this paper will discuss how an awareness of the unusual characteristics of the '*Public Land Survey System Datum*' and the historical '*Manual*' procedures lead to some interesting conclusions about the proper way to compute double proportion positions for lost corners. This discussion will include examples of how both large and small errors can creep into the process when using coordinates, especially State Plane Coordinates, and point out dangerous situations as well as proper methods to use to avoid pitfalls.

## GENERAL

This presentation deals with one of those seemingly insignificant technical issues that may rise up and bite you if you are not careful. This particular discussion relates to the procedure used to restore certain lost corners in Public Land Surveys by the process called double proportion. This is a well known procedure to surveyors in such public land states who practice in suburban or rural areas. It is a process that seems straightforward on the surface, but is also sometimes misunderstood and incorrectly computed.

In defense of this *technical* presentation, I would like to point out that in my opinion there are many aspects to the profession of surveying. Some of the aspects on which we place the highest importance are the evaluation of evidence, discovery and analysis of prior records and application of judgement. These and other professional issues are well recognized and are significant issues in licensure. Somewhere on the list of attributes that constitute the makeup of the profession of surveying is technical expertise and knowledge of proper procedures in measurement and computation. By no means are these considerations primary, but neither are they insignificant. This discussion is almost entirely technical and one sided. Whereas in the real world I recognize that many other factors control our actions and considerations.

Before I can illustrate some of the technical quirks of double proportion, I need to briefly describe something that I refer to as 'The PLSS Datum'. This datum is simple but has some unique and even strange attributes. A thorough description of it and all it's consequences could easily be the topic of several papers, so what is outlined here is necessarily brief.

## THE PLSS DATUM

The 'PLSS Datum' is the reference system by which the majority of the PLSS surveys are theoretically reported. The data being reported on a BLM or GLO Cadastral Survey plat are, of course, bearings and distances. But bearings and distances with reference to what? The current BLM Manual of Surveying Instructions, 1973 states:

"2-1. The law prescribes the chain as the unit of linear measure for the survey of the public lands. All returns of measurements in the rectangular system are made in the true horizontal distance in miles, chains and links...."

"2-17. The direction of each line of the public land surveys is determined with reference to the true meridian as defined by the axis of the earth's rotation. Bearings are stated in terms of angular measure referred to the

true north or south."

"2-74. .... By basic law and the Manual requirements, the directions of all lines are stated in terms of angular measure referred to the true north (or south) at the point of record."

**Distances.** These and other references in the BLM and GLO Manuals make it clear that the frame of reference for *distances* is defined as horizontal measure in chains based on the U.S. Survey Foot at actual ground elevation. This is of importance when performing computations in projections or at sea-level when the actual lines are at a significant elevation. If you are computing proportions in a projection, the variation of elevation over a project can have a small effect, the elevation difference in essence weights the record ground measurements. This usually is a small effect unless the lines differ in elevation by a 1000 ft. or so.

**Bearings:** The above Manual sections and others identify the frame of reference for *direction* as something called 'Mean True Bearings' referenced to the true astronomic meridian '...at the point of record.' For those of you familiar with basic geodesy you will recognize that this is a basis of bearing that changes as you go east and west since the reference meridians are not parallel but converge towards the pole.

Because this is a changing reference, the direction of a straight line on the ground can be described with a forward bearing based on the meridian at the beginning end, or with a differing back bearing based on the meridian at the end point. The difference between them is the angle of convergency of the two meridians. If we want to accurately describe how far north or west the line goes in a geodetic sense, we need to use the average or 'mean' of these two values. This '*mean bearing*' is essentially identical to the bearing of the traverse line with reference to it's midpoint. Thus the 'point of record' for determining the bearing of a straight traverse line can be said to be the meridian at the midpoint of the line.

**Straight Lines:** Therefore, one unusual byproduct of the PLSS datum is that:

***Straight lines on the ground are lines of constantly changing bearing.***

A straight line is basically what you would lay out by double centering or projecting a direct line of sight. The only straight line that does have a constant bearing is the meridian or north and south line. An example of a boundary that might be a straight line is one that is described as a straight line running from one physical monument to another. Such a line, if reported in the PLSS Datum would have different forward and back bearings, and different bearings at each point along it.

**Rhumb Lines:** It is also apparent from the various GLO and BLM Survey Manuals and the actual methods that were used to lay out the public land surveys that most boundary lines in the PLSS are intended not to be straight lines but lines of constant bearing or Rhumb Lines. Such lines cross every meridian at the same angle and are thus curved as viewed on the ground.

Therefore, another unusual byproduct of the 'PLSS datum' is that:

***Lines of constant bearing are curved lines on the ground.***

For example, the solar compass and transit were instruments that determined bearing at each setup, and when matched with traditional chaining, measured or laid out lines of constant bearing.

The '*Manual*' discussion of latitudinal arcs illustrate one example of a rhumb line. A parallel of latitude is a line that is due East and West in the PLSS Datum. Since it crosses each meridian at a 90 degree angle, it has a mean bearing of East or West. Lines of constant bearing in the PLSS datum will appear curved on the ground. It also turns out that the mean bearing of any chord or sub-chord connecting any two points along such a line is the same as the bearing of the rhumb line itself. Thus it is possible to lay out points on a rhumb line by correcting traverse lines to their mean bearing in computations.

## DOUBLE PROPORTION

Now let's look at the definition of double proportion as stated in the BLM *Manual of Surveying Instructions, 1973*, which states:

"5-25. The term 'double proportionate measurement' is applied to a new measurement made between four known corners, two each on intersecting meridional and latitudinal lines, for the purpose of relating the intersection to both.

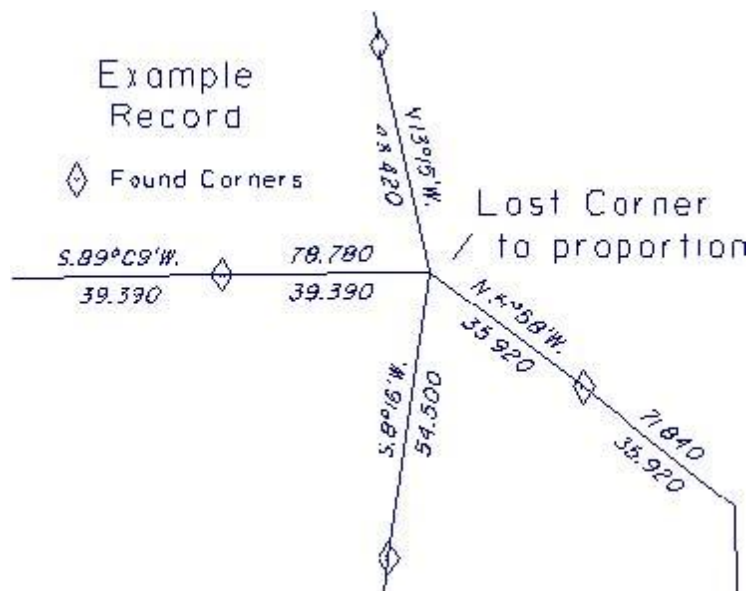
In effect, by double proportionate measurement the record directions are disregarded, excepting only where there is some acceptable supplemental survey record, some physical evidence, or testimony that may be brought into the control. Corners to the north and south control any intermediate latitudinal position. Corners to the east and west control the position in longitude."

.....

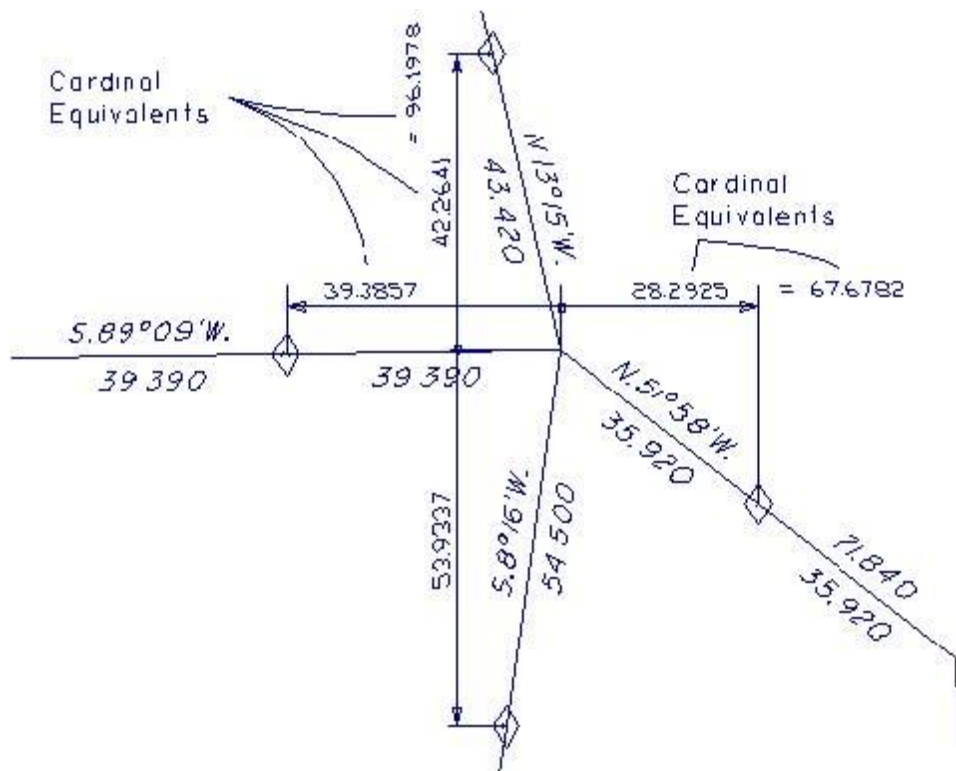
"Lengths of proportioned lines are comparable only when reduced to their cardinal equivalents. "

**Cardinal Equivalents:** The last sentence in the above quote is one that requires some explanation. What it means is that only the easterly components (or departures) of the E-W controlling record lines are used to compute the E and W position, and only the northerly components (or latitudes) of the N-S controlling record lines are used to compute the N and S position. This is different than using the line lengths or distances on the record line.

Figure 2 illustrates the cardinal equivalents for some of the lines in the example record shown in Figure 1. Neglecting to correct the record for cardinal equivalents won't usually get you in trouble since most section lines in the original surveys are very near to cardinal and the correction is insignificant. There are, however, many situations in public land surveys where this is not the case. This situation will also occur where a retracement or subsequent GLO or BLM resurvey has reported new measurements in the PLSS datum, and the lines are distorted. The record shown in Figure 1 is a typical example, and is used to illustrate the problem.



**Figure 1 - Example Record**



**Figure 2 - Cardinal Equivalents**

### Cardinal offsets

*The Manual of Surveying Instructions, 1973* Section 5-26 describes a process for performing a double proportion. The Manual section states:

"5-26. In order to restore a lost corner of four townships, a retracement will first be made between the nearest known corners on the meridional line, north and south of the missing corner, and upon that line a temporary stake will be placed at the proper proportionate distance; this will determine the latitude of the lost corner.

"Next, the nearest corners on the latitudinal line will be connected, and a second point will be marked for the proportionate measurement east and west; this point will determine the position of the lost corner in departure (or longitude).

"Then, through the first temporary stake run a line east or west, and through the second temporary stake a line north or south, as relative situations may determine; the intersection of these two lines will fix the position for the restored corner."

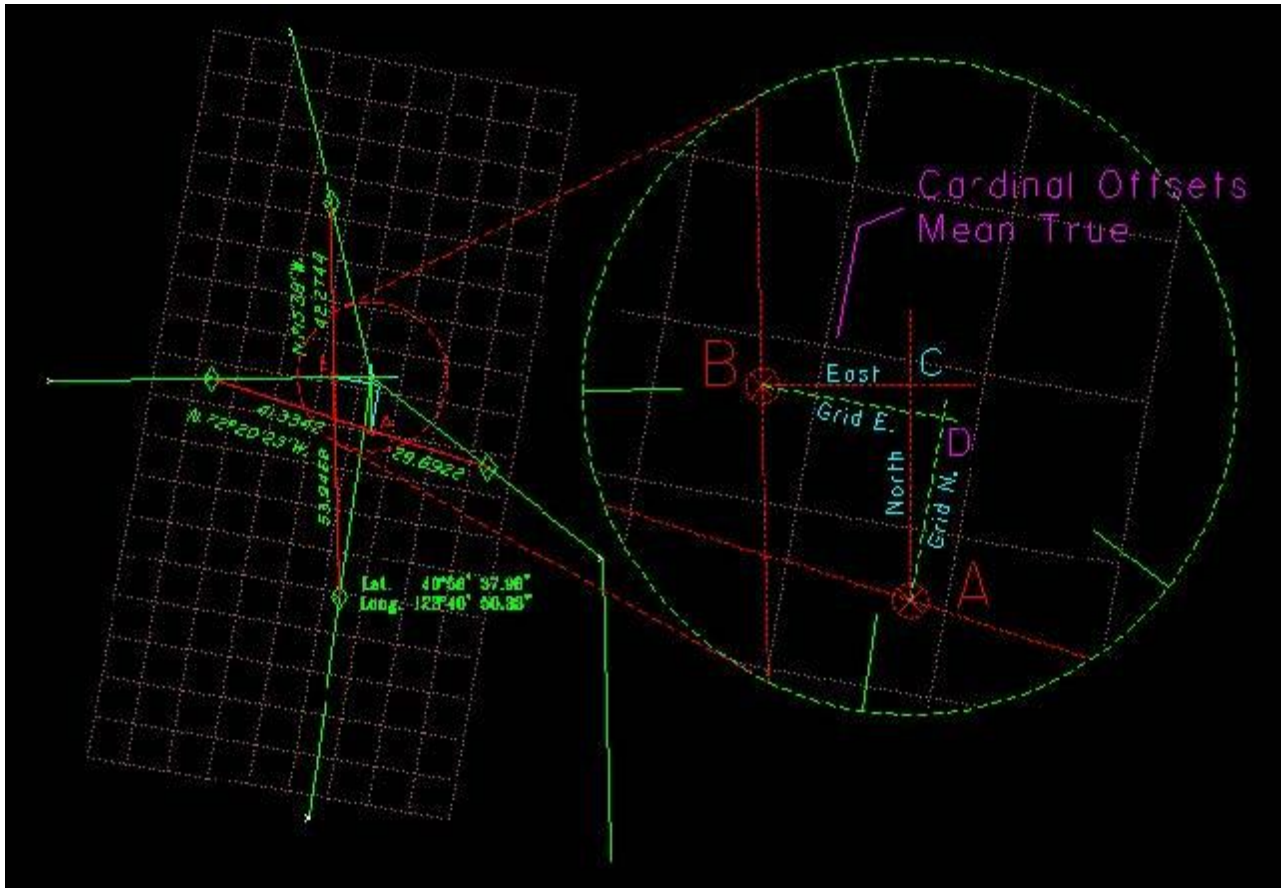
Such a process would probably be impractical in the field if followed to the letter. It is, however, a valuable way to conceptualize a proper solution of a double proportion and a good way to model a computational method.

In brief, the three part process as described consists of:

- 1 A single proportion using the record E-W cardinal equivalents between the control E and W. In the Figure 3 example this would be point 'A'.
- 2 A single proportion using the record N-S cardinal equivalents between the control N and S. In the Figure 3 example this would be point 'B'.
- 3 Cardinal (true mean) offsets to intersection from those two points. In the Figure this results in point 'C'.



This last requirement can be a problem if you are not careful using coordinates, since to make the offsets cardinal requires knowledge of and proper correction to true north at those points. The common process of using the East coordinate of the E-W proportion and the North coordinate of the N-S proportion is equivalent to making a GRID offset, exaggerated in Figure 3 as point 'D', which can be incorrect.



*Figure 3 -Cardinal Offsets True vs Grid*

### Problems with State Planes.

In using state plane coordinates there are several small problems, however dealing with cardinal offsets is the most critical. This is true because with State Planes there can be a large difference between grid and true north, which is called the mapping angle. There are three methods that can be used to correct for cardinal offsets using State Plane coordinates, they are:

- Compute the mapping angle at point A to find what grid azimuth equals true North, then compute the mapping angle midway between A and B to find what grid azimuth equals mean East, then compute a grid bearing intersect. If the offsets are not large, one computation of mapping angle in the area will be adequate.
- Convert the coordinates to latitudes and longitudes at points A and B, use the latitude of B and the longitude of A and convert back to State Plane. This method essentially makes a geodetic cardinal offset.
- Convert the coordinates of the control corners to latitudes and longitudes and proportion them in a similar way to the above.

Small errors can still exist in the computation due to the datum differences. In State Planes the grid scale factor varies over the project. If you wanted to be perfect, this would require you to weight the proportions according to the mean scale factor over each line. This effect is very small. The last method is the easiest, since no correction for scale factor enters into the problem. However, performing proportions using geodetic coordinates directly can still have error if the lines are at very different elevations, since the PLSS datum represents measurements at actual average ground elevation

over the line.

In the example problem, the error caused by direct proportion of the State Plane Coordinates is in excess of 20 ft. If you used a local grid or basis of bearings the following table illustrates corresponding errors for this example.

Grid Angle Error D to C

1° 05' 20"	20.11 ft.	California Zone I
0° 10'	3.40 ft.	Assumed 10' off true.
0° 01'	0.65 ft.	Assumed 01' off true.
0° 00' 45"	0.46 ft.	Basis of Bearing, Solar 1 mile E.

## SUMMARY

It can be shown that in an ideal world and with a recognition of the properties of the *PLSS Datum*, the only way to properly restore a corner in its true original position is by diligent application of the *Manual* procedures, correcting record to cardinal equivalents, proportioning and making true cardinal offsets.

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