Course 4: Restoration of Lost Corners Study Guide

COURSE DESCRIPTION:

This course consists of four videos, some reading, and three exercises, on the “Restoration of Lost Corners”. The legal, mathematical, and practical applications of the methods of proportioning, as found in the Manual of Surveying Instructions, are presented. Students will be able to address what corners control in most situations, how to proportion properly, what legal principles are involved when proportioning, and how to deal with the latitudinal curve. A lengthy discussion of convergence and curvature in the PLSS is also included.

COURSE OBJECTIVES:

Upon completion of this course, students will be able to:

- Define the three corner conditions listed in the Manual of Surveying Instructions
- Describe, identify applicability, and compute proportions using all methods
- Demonstrate an understanding of curvature in the PLSS

COURSE INSTRUCTOR(S):

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VIDEO LECTURE TITLE:

Restoration of Lost Corners – Part 2 (42 minutes)

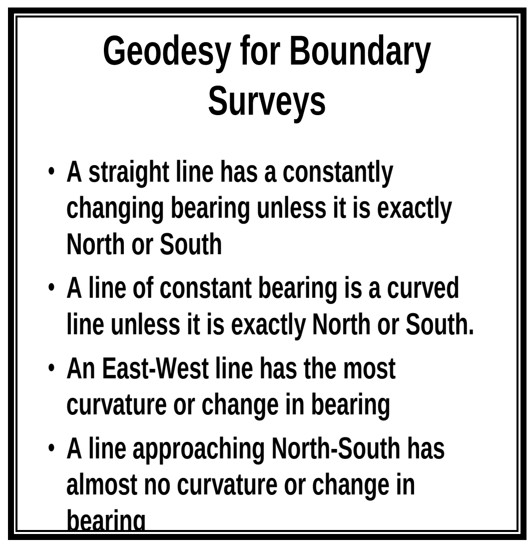
ICON LEGEND							
WEB COURSE	EXERCISE	DIAGRAM	READING ASSIGNMENT	PROBLEM	HANDOUT	2009 BLM MANUAL	QUIZ

RESTORATION OF LOST CORNERS – PART 2

Introduction

Welcome back. First of all I would just like to look at a couple of principles, a couple of issues we have to deal with when we are working on the surface of the earth when we are trying to work with true bearings.

And I think these will help us maybe understand exactly what is going on here.



Now first of all a straight line has a constantly changing bearing unless it is exactly north south or of course if its on the equator. But a straight line has a constantly changing bearing, it is a curved line as far as bearing goes.

It is a straight line, even if we have line of site, a straight line the bearing is constantly changing unless it is exactly north south or at the equator.

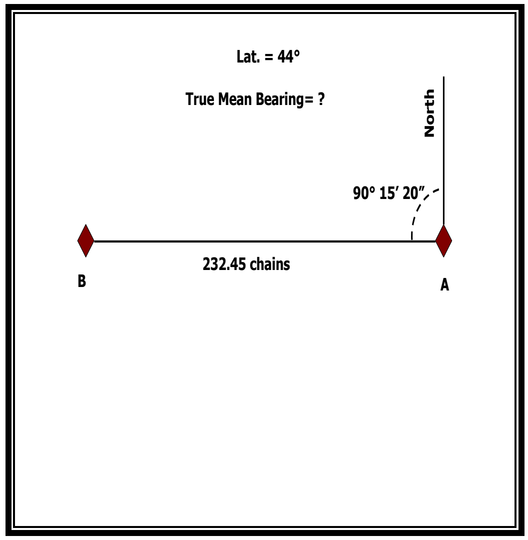
A line of constant bearing is a curved line, unless it is exactly north south. So if we want a line of constant bearing, which is what we find in the Public Land System, if we want that, it is a curved line. If we have a straight line it is a line of constantly changing bearing. That is a real key issue here.

An east west line has the most curvature or change in bearing as it approaches north south, it has less and less and less change because it is a function of its departure.

Lines approaching north south have almost no curvature or change in bearing because there is very little departure and it is a function of departure.

RESTORATION OF LOST CORNERS – PART 2

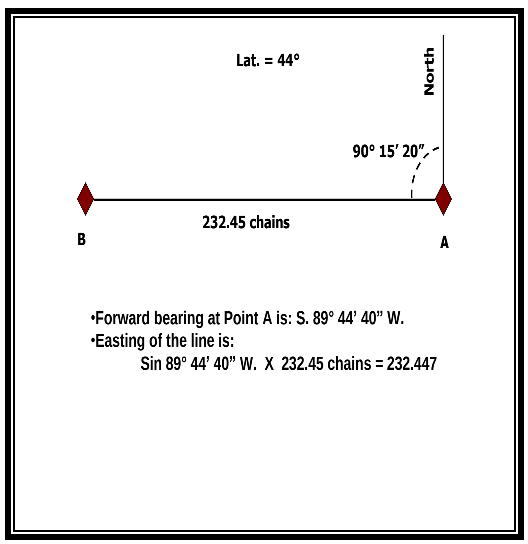
First of all let's just look at a very simple example and see how we work through and deal with this curvature, how we get a true bearing of a line on the surface of the earth and how that affects our corners, how that affects those things.



First of all let's look at this one, Latitude 44 degrees of course the closer we are to the equator, the less change there is, the less convergency, the less curvature, the closer we are to the pole, the more curvature, the more convergency. True bearing. How do we come up with the true bearing here?

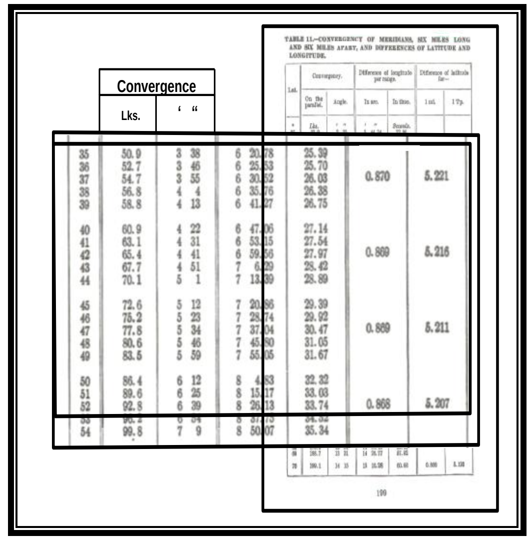
So let's assume that over here at point A that line to the north is true north. We have that bearing, we've established that, we have turned an angle and we have a distance. So with that angle we can compute that our forward bearing at least at point A is south 89° 44'40" west, simply the angle from north. 89° 44'40" south west. That is our forward bearing at point A.

Now, the easting of the line, the departure of the line, we can find by the sign of the bearing times the distance, 232.45 chains and that will give us the departure of the line and of course the departure is very close to the distance because this line is very close to north south.



So we end up with a departure of this line AB is 232.447 chains. That is the departure of our line. We have a forward bearing. We know that it is a straight line.

RESTORATION OF LOST CORNERS – PART 2



Let's say that we can sit with our instrument on point A and we can see point B so we know that it is a straight line, we know what the forward bearing is and now we know what the departure of that line is.

So we go to the standard field tables and that is contained in your **resource disc** in your set of DVDs, the standard field tables, a lot of interesting information in there and where we want to go is to Table 11, and Table 11 is convergency of meridians, six miles and six miles apart and differences of latitude and longitude.

Well, meridians six miles long and six miles apart, well what we are talking about here is a township really and that helps us think about what this table is giving us actually It is talking about a township. And it gives us various pieces of information about this township for various latitudes.

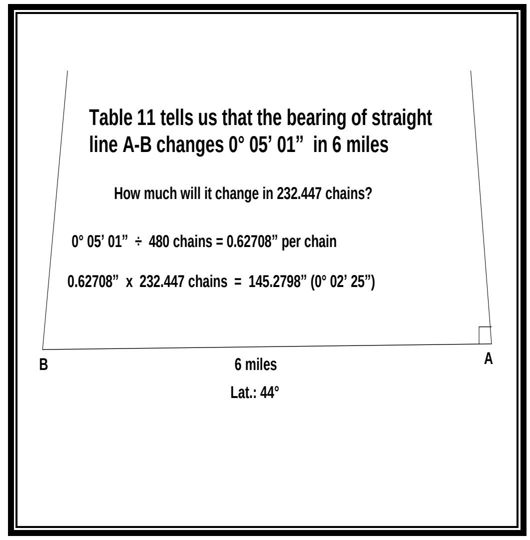
It gives us convergency, difference in difference in longitude difference in latitude and the convergency is on the parallel or on the angle.

So let's go and look at our table for the situation that we are dealing with which is 44 degrees. So at 44 degrees, we are going to go to our table, we are going to go to our table and right here we are going to find that the convergency is 70.1, but that is not really what we are looking for that is talking about linear convergences.

So we want to go to the next table, the next line and it tells us that the meridians which are six miles apart remember, they converge zero degrees five minutes one second at this latitude, at 44 degrees, meridians that are six miles apart are going to be converging by five minutes one second.

RESTORATION OF LOST CORNERS – PART 2

So what does that tell us. Table 11 tells us that the bearing of a straight line AB is going to change zero degrees five minutes and one second in six miles.

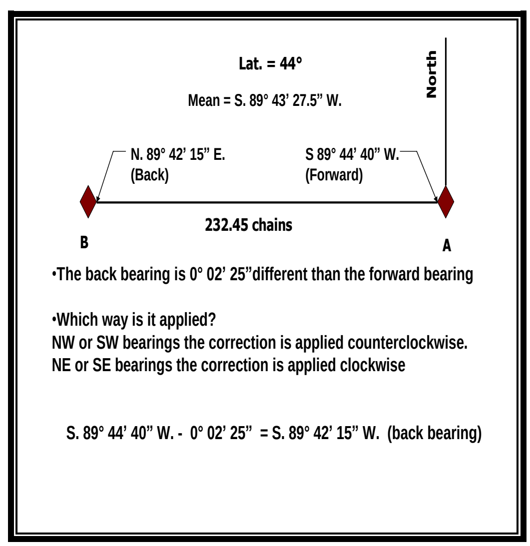


Because those meridians are converging. So our straight line bearing is going to change five minutes and one second in six miles. Well, so how much will it change in 232.44 chains because that is the departure of our line AB.

Well, if we know it is going to change five minutes and one second in 480 chains, that is six miles, 480 chains equals six miles and these meridians are six miles apart in our table, so we divide five minutes and one second, that is 301 seconds by 480 chains and that gives us the factor .62708 seconds per chain so that is how much our bearing is going to change per chain of departure.

So for every chain of departure at 44 degrees, our bearing, the bearing of a straight line is going to change 0.62708 seconds.

Let's go on then. So for our line which is 232.44 chains long, we multiplied the factor time the departure of our line, line AB, which gives us 145.2798 seconds or 2 minutes 25 seconds.



So on line AB we have just computed that this straight line that began at point A, at point B that bearing has changed 2 minutes and 25 seconds. So the back bearing at B, so the bearing at B looking back is going to be 2 minutes 25 seconds different than the forward bearing at A which was 89°44'40".

But which way is it going to be applied? We go counter clockwise, clockwise? Are we going to add it? Subtract it? What are we doing here? Well there are a couple of rules.

There is a couple of ways of stating it. First of all north west and south west bearings the correction is applied counter clockwise. So if you have a line that is exactly west, north west or south west, the correction is applied counter clockwise.

If you have north east or south east bearings, the correction is applied clockwise and that includes bearings that are exactly east. So any line that is going easterly to any extent, it is clockwise. If the line is westerly, it is counter clockwise.

RESTORATION OF LOST CORNERS – PART 2

Now another way to look at it is if it is a northerly bearing, north east, north west, the correction applied is added. If it is south east or south west, the correction is subtracted and if it is east or west exactly, the correction is subtracted. Just another way to look at it.

We have a couple of rules there to help remember which way we are going to apply this correction. Because it can get confusing, but once you get it in your head exactly what is happening, you kind of get a picture of it, it is much easier to apply that rule or that correction in the proper method.

So if we look at our diagram, we can see south 89°44’40” west minus because what do we have, we have a south west bearing, it is a south one so we subtract it or it is a westerly one so we go counter clockwise. So either rule works here. It is just however you want to look at it.

So back to our diagram, we can see that 89°44’40” south west minus 0.0225 seconds that’s our correction equals a bearing of south west or since it is a back bearing, now it is north east 89° 42’15”. That is our back bearing. We now have a straight line between points.

We have a forward bearing and a back bearing, so it is a simple matter to get a mean bearing. And that comes about by just averaging those two and we end up with a mean bearing of 89° 43’27.5”. So what is that telling us?

That is telling us that when we establish corners on this line, this line is going to have several quarter corners and section corners in this 232 chains that we can’t put them on a straight line because they were established on a true bearing across there. We need to establish them on this curved line.

They need to be on a line that has a constant bearing of 89° 43’27.5” and as we discussed earlier, a line that has a constant bearing is not a straight line it is a curved line. So we need to find a way to do that and I am going to show a couple of examples, two different ways, and there are actually others.

One way to go about this is to run a tangent and the Manual talks

RESTORATION OF LOST CORNERS – PART 2

about running a tangent line and the Manual talks about running a secant line. There are several ways to go about this. I am going to give you a couple of examples of how to account for curvature in your calculations and I think these work pretty well for your day to day surveying.

Also of course there are also programs out there that take care of geodesy and deal with the curvature that you may be familiar with and that you may be using. But it is important that we have a good understanding of what is actually happening so that we can catch errors, so we make sure that we do it right and we really know what is going on.

So let's look at this and let's assume that point a, b, the small a, small a, b, c. d and e are temporary points.

So this is a situation, this is probably something that is going to happen maybe you are surveying in the northern California coast and it is pretty brushy, pretty nasty country and you have to actually survey this on the ground with a traverse on the ground, so you are setting a temporary point somewhere near where you believe the corner is going to end up so you have a point at 39 85 5, one at 80.024, one at 120.107 and so forth.

Now for simplicity I have these all on a straight line, these are all on a straight line. Most of the time that is not going to happen, you are going to have some bends. But I think you will see that it's the same process is used even if you have several traverse points between these temporary points.

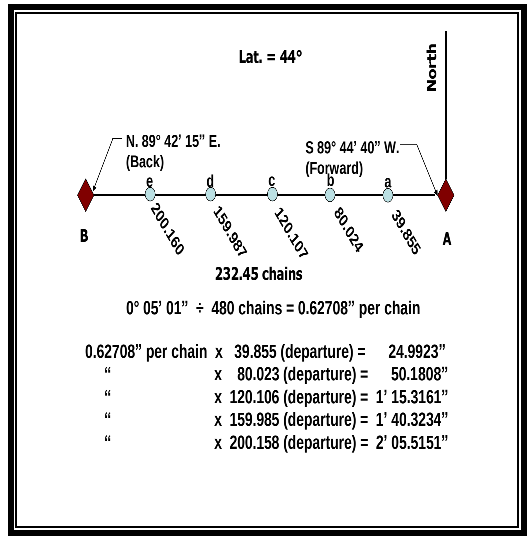
So we have a total distance of 232.45. We' re at latitudes 44 degrees and we have these temporary points, we have a forward bearing. We have a back bearing. So let's see what we can do here. Well, first of all remember we had this factor that we got by dividing five minutes and one second which is the curvature and meridian six miles apart at 44 degrees, it comes from Table 11 in the standard field tables.

We divide that by 480 chains, again which is six miles in chains and we come up with a factor, curvature per chain in seconds 0.62708. Well, let's look at this first course. If we want to compute the true bearing of this straight line at point a we can take

RESTORATION OF LOST CORNERS – PART 2

0.62708 our factor per chain times the departure in chains between point a (small a) that's the departure and we end up with 24.9923 seconds.

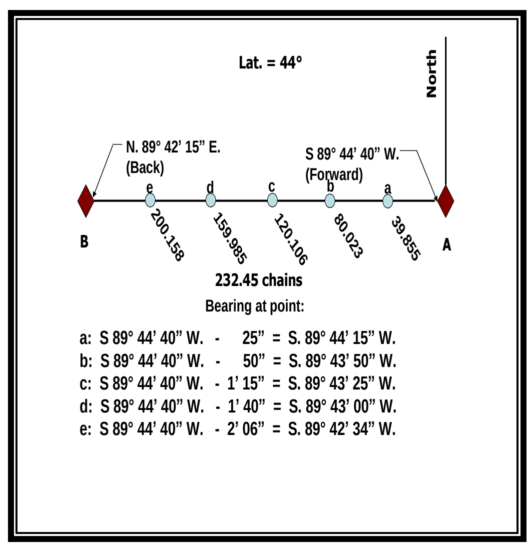
So in that half a mile basically, this line, this straight line of ours, the bearing is going to change 24.9923 and we just move on down here to the next point the bearing is going to change 50.1808.



At the next point the true bearing of the line by the time we get to point c on our straight line, the true bearing has changed one minute 15.3161 seconds. So we can see as we go across that bearing keeps changing. We keep going you know to point d and finally over here at point e. The departure there is two minutes .5151 seconds. So now we know what the correction or the curvature of that line, the change in bearing of that line is as we go across.

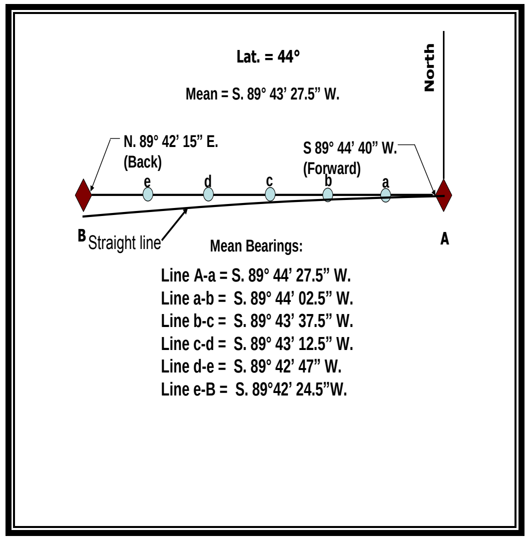
Beginning with our forward bearing of 89°44'40", we know that at point a, that bearing has changed 24 seconds, at point b it has changed 50 seconds, at point c a minute and 15 seconds. It is telling us how much that bearing is changing as we progress across there with this straight line. So from that we can then compute bearings.

So the bearing at point a, now remember we have a south west bearing. It is westerly, so the correction is applied counter clockwise, so we need to subtract it. 89°44'40" minus 25 seconds equals south 89°44'15" west. That is our mean bearing there. Keep going same thing.



Subtract the 50 seconds, now we have the mean bearing at point b. Same thing. Subtract the minute 15 seconds, we have the mean bearing at point c and so forth, point d, point e. And now how do we come about computing the mean of the courses because we don't just want the mean at point a or at point b, we want the mean bearing of line aa, ab, bc. So let's look at these. Here is the mean bearings now.

RESTORATION OF LOST CORNERS – PART 2



If we have the bearing at each point, all we have to do is mean them to get the mean bearing of the line. So the mean bearing of line Aa is 89°44'27.5", Ab is 89°44'02", Bc is 89°43'37" and so on. Now what do we have here? We have temporary points that we have established.

We surveyed them on a straight line. We have now computed true bearing between each of those points. So we have a mean true bearing for each course across this traverse, between these points. You are done. That is all you need to do to account for convergency.

From this point you can completely forget about the convergency. All you need to do is use the bearings and distances that you have computed now and you can compute your corner moves the way you always do. You can do your proportionings the way you always do.

From now on once you get to this point you can forget about convergency. It is accounted for it. You've accounted for it when you put the mean bearings on each of these courses. You've created a curved line across there.

Now as we'll talk about in a little bit. There is one place that you have to still think about it and that is the linear convergency because any time we use true bearing, figures do not close, we'll get back to that in a minute but for this calculation if you were going to single proportion all the corners along this line, let's say if this is the south boundary of a township and we have these five missing corners and we are single proportioning these five missing corners, you have taken into account convergency.

All you have to do now is a single proportion using these bearings and distances and you are done. It is taken care of. All right, so let's take a look at a different method now.

So if we have a single proportion along this line and we need to reestablish these five lost corners, we have already accounted for convergency, we have accounted for the curvature I mean, and all we need to do is a normal single proportion using the numbers that we get from these bearings and distances. It is done. You have taken care of it.

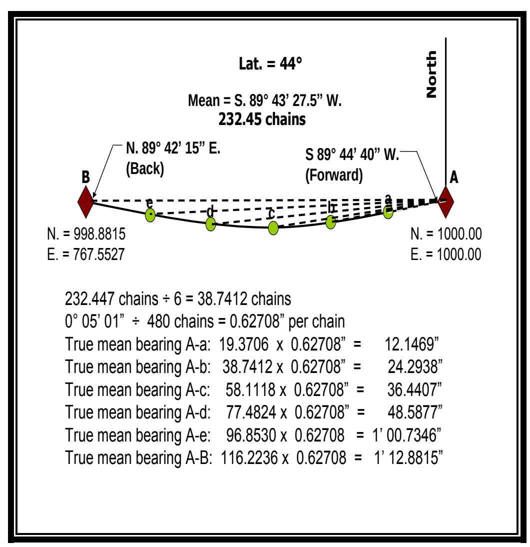
RESTORATION OF LOST CORNERS – PART 2

You have a mean bearing now of 89°43’27.5” for the entire line. That is our mean bearing and I just wanted to show exactly what is happening here. The straight line, it is straight but its bearing is changing and that is what is represented by the line that I just put on here.

It says it is a straight line, but actually it is but its bearing is changing, its bearing is curving. Its bearing off this way is changing counter clockwise. It is rotating counter clockwise. I hope this picture helps. Picture in your mind a little bit what is happening there. Now I want to look at another example which is a different method for computing this and this would be a situation where we have coordinates on either end of the line.

So here we have coordinates on each end we do not have temporary point in the middle. And what we are trying to do is we are trying to calculate the position of each of these lost corners along this line and put each of these corners on the curve and again, we have the same forward bearing and back bearing, we’ve computed the mean bearing, we got the same distance and we have a computed departure here. So let’s see how this method changes a little bit and there are a couple of things that are different and they are important differences.

First of all, we have to proportion. Right? Because we are doing a single proportion here so we have 232.447 chains. Remember that was the departure of this whole course, so we are going to divide that by six, because we have six equal segments across here. Six half miles between original corners and it equals 38.7412 chains per half mile.



These originally were 40 so there is some error in here. So now we know that we want to establish a point, we want to begin at point a and we want to go at a mean bearing of 89°43’27.5” that is our bearing south west 38.7412 chains. So how do we do that? Well, we need a point at each of these.

Let’s start by same thing, zero degrees five minutes one second divided by the 480 chains gives us the .62708. That is our change per chain. We need that again. So now we want to calculate the true mean bearing Aa and that is not exactly what we are doing but

RESTORATION OF LOST CORNERS – PART 2

let's think about this.

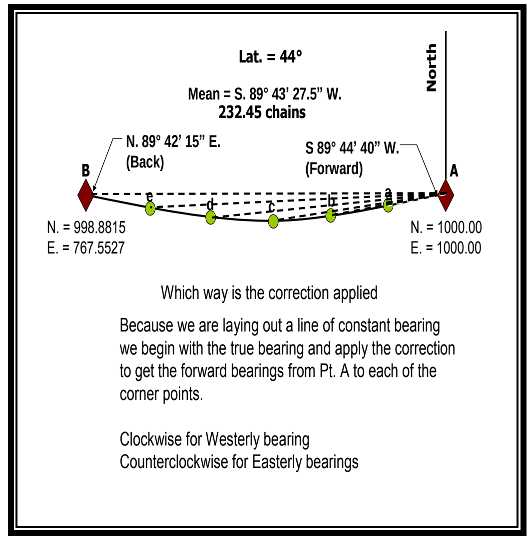
We have already computed what the mean bearing of this entire line is so corners a, b, c, d and e all want to be on that mean bearing. So we are going to take half of line Aa times our factor and that is going to give us the correction we need to get from a forward bearing to a bearing on the true mean bearing, which is the curved line. So from the forward bearing to the mean bearing on line Aa ends up being 12.1469 seconds.

Now we used half of the distance to the line because we are not trying to calculate how much that line the bearing changes in that half mile, we are trying to calculate the mean of that line. So we only want the chain for half of it. At the midpoint. That is how we are going to get the mean. So this is the change in bearing of a straight line Aa. And what this actually is is it is the angular difference between our mean true bearing and a forward bearing to point A. And we will look at that again a little bit more so that we are certain that everyone understands that. So let's look at the next one, b, 24 seconds and again we go 38.7412 chains. That is the midpoint of line Ab. Capital Ab. The midpoint of that line times our factor gives us 24.2938 seconds. That is the angular change between the true bearing which we know, remember we calculated the true mean bearing of this line, 89°43'27.5" seconds.

We want to go from that true mean bearing to a forward bearing that will take us from point A, Capital A, to point b. Let's look at the next one. Again, midpoint of line Ac times our factor gives us 36.4407 seconds. That tells us how much curvature we've got across there and we are trying to get now a forward line from A to C.

We already know what the true mean bearing is, the curved line, we are trying to figure out what angle do we need to turn now to get a forward bearing that will take us here and it is it's going to change 36.4407 seconds. And you will see how this works as we go along. We go on for point Ad, Ae, same process and the last one clear across.

RESTORATION OF LOST CORNERS – PART 2



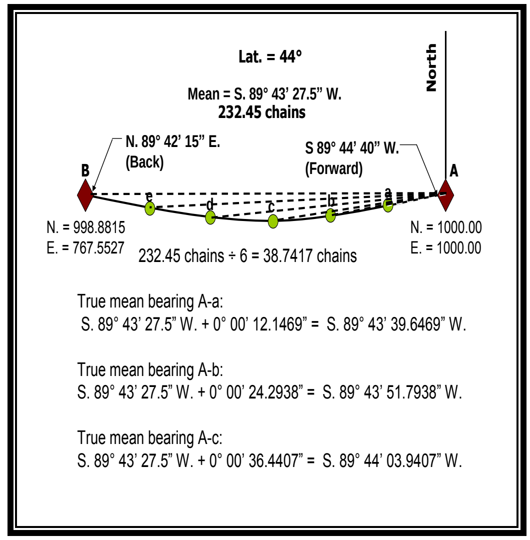
Now let's look at the next step and see what happens. First of all which way are we going to apply this. Now remember that when we had a forward bearing, a straight line and we calculated this factor, this curvature for each segment of the line, we said that we applied it counter clockwise, if it was a westerly bearing and clockwise if it was an easterly bearing. In this case, we are not going from the straight line to the true mean bearing.

We are going from the true mean bearing back to a forward bearing, the straight line. We are going in the opposite direction. We're going to use these numbers that we created to go from the true mean bearing to a forward bearing so that you can calculate where these points go from your coordinates. So let's look at what happens.

Because we are laying out a line of constant bearing, we begin with a true bearing and apply the corrections to get to the forward bearings from point A to each of the corner points. That is an important factor. So we are going to apply these corrections in the opposite direction. We are doing a different thing. We are laying out a line here. We are laying out a line.

In our previous example, we had points on the ground that we had measured between and we were trying to calculate what the mean bearing was through it. Here we are doing the opposite. Clockwise for westerly. Counter clockwise for easterly. The opposite of the example before. So let's look at how this works. Again here are our coordinates, here is our diagram, forward bearing, back bearing, mean bearing.

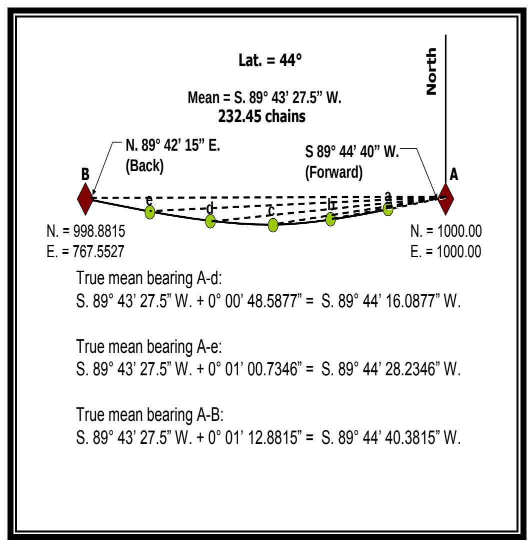
RESTORATION OF LOST CORNERS – PART 2



Here's what happens. Again our proportions 38.7417. Here's our corners. Now south 89°43'27.5" that is our true mean bearing. So what we want to do is remember we are going south west, so we are going to go clockwise. We are going to add our correction which is 12.1469 seconds and that gives us a forward bearing from point A of 89°43'39.6469".

That is what we need to lay off on the ground at point A we need to turn an angle from that north line that will give us 89°43'39.6469". That is the forward bearing to get us to point A. From Ab we do the same thing except that now we add the 24 seconds. And you notice we get a bearing of 89°43'51". That is the forward bearing at point A that will give us this corner point, this true corner point for b.

Where we want to set the corner. This proportion position. We do the same thing for c. We add the factor that gives us a new bearing.



I want you to see what happens now as we continue across here and we go a, b c, d and e and on across. Now we will go to this next slide and we will see. Watch what happens with line Ad. Again we add the correction.

We add it to the mean bearing and that gives us a forward bearing to get from A to d and we do the same thing with Ae. Again we add it. Look what happens. When we add that correction from the mean bearing we add the correction and we get south 89° 44'40.3815" seconds west. Look what our forward bearing was to begin with 89°44'40".

Of course we rounded it off but we match. That is the same bearing. All we have done is we have computed the opposite direction now. We have computed from the mean bearing to get to the forward bearing so that we can lay this off because what we were doing before we had already set monuments and we were working to compute where those temporary corner points were.

Now we were trying to compute where to set those points and of course in this case, they are not temporary, we have computed where the true corner point goes. Where the proportion corner

RESTORATION OF LOST CORNERS – PART 2

point goes is on these forward bearings and at the correct distance.

So to get the coordinates let's just work through one example to get the coordinates of point c. What would we do? Well, we know the departure of c is 116.2237, you can check back and you will see that that is correct. We know the sign of our forward bearing that we computed is 89°44'04", we know that that is the forward bearing so we divide the departure by the sine of the bearing and that gives us the distance of Ac which comes up right here 116.2249 very little change.

Calculate the coordinates for corner C:

116.2237 ÷ sin S.89° 44' 04" W. = Dist. of A-c
116.2237 ÷ 0.999989 = 116.2249

cos S. 89° 44' 04" W. x 116.2249 = 0.5387
sin S. 89° 44' 04" W. x 116.2249 = 116.2237

N. 1000.00 - 0.5287 = N. 999.4613
E. 1000.00 - 116.2237 = E. 883.7763

So then we can take the co sine of the bearing times the distance and that gives us the north or south component, here we have a south west line so it gives us the southing of that line basically.

We take the sign of the bearing times the distance, the length of the line and that gives us the west component. So now we know from point A this is the coordinate that we want to set our monument and we have taken care of the convergence here, and now we get to the coordinate, we have to add those of course to our initial coordinate, add or subtract, and we do that and we end up with the coordinates of point c. We don't have to think about curvature anymore. We have taken care of curvature now.

Now we are just back to just surveying plain. We have computed these forward bearings, all we have to do is turn these forward bearings. We have got our bearings at A established from point A we have computed forward bearings and distances to each of these points.

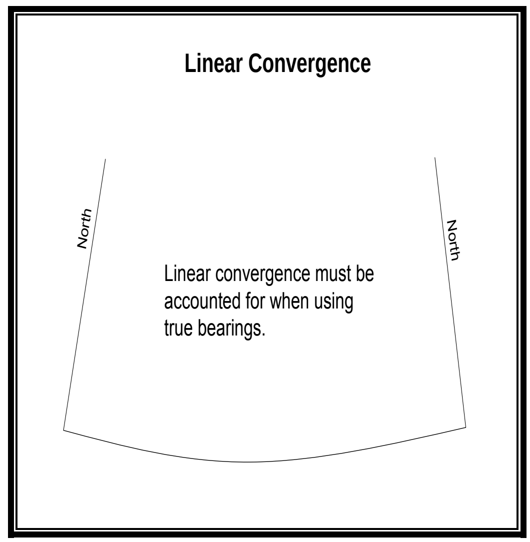
We can compute coordinates on each of these points and we can go out on the ground and set the monument at those points. And they will be on a curve. They will not be on a straight line. They will be on a line of constant bearing on a straight line. Two different methods, one when we have set temporary points to begin with.

How we deal with curvature. Number 2 when we have coordinates on the controlling corners and we need to calculate corner points within. Pretty basic. It is not high level geodesy. But it is well within the accuracy that we need for these boundary surveys. It is pretty straight forward.

RESTORATION OF LOST CORNERS – PART 2

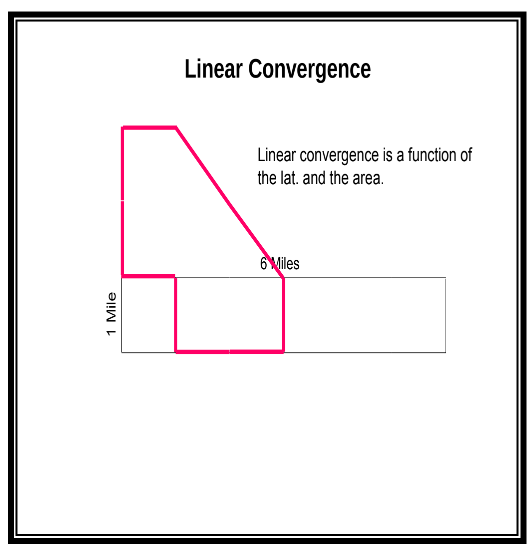
It is not very difficult to really grasp once you work with it a little bit. To begin with it is a little bit confusing because straight lines aren't really straight and curved lines aren't really curved and all that kind of thing. But once you work with it a little bit, I think that you will see that either one of these methods are pretty easy to work with and it is pretty easy to program a calculator to do it and if you are using some other kind of program that already has curvature and geodesy built into it. Wonderful. Hopefully, this just helps to understand exactly what is going on.

Now there is one more thing that we need to look at and that is linear convergency because we know that if we use true bearings and distances to calculate around a closed figure, it is not going to close, even if all of our measurements and bearings and angles are exact.



It will not close because meridians converge. So we need to talk about that and we have to account for any time we are using true bearings.

So the linear convergency we have to deal with and we will just look at an example here and it is a function of latitude okay because we have more convergency of meridians the further north we go so it is a function of latitude but it is also a function of area.



It doesn't really matter what shape we are dealing with, it is area. So it doesn't matter if we have a shape that is six miles by one mile or if we have a shape like this red figure here that contains six square miles also each of these areas contain six square miles therefore, and they are at the same latitude, therefore they are going to have the same linear convergency.

RESTORATION OF LOST CORNERS – PART 2

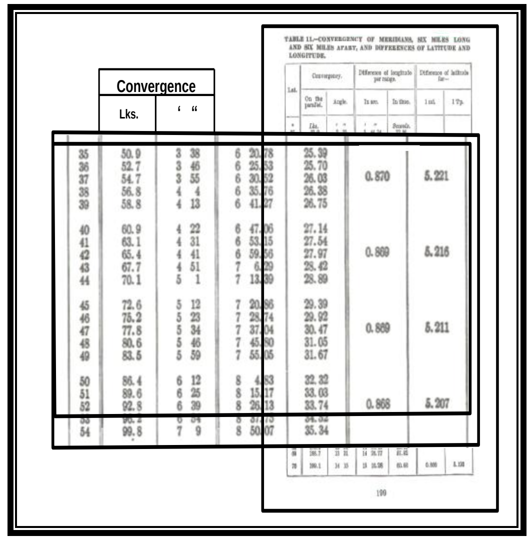


TABLE 11.—CONVERGENCY OF MERIDIANS, SIX MILES LONG AND SIX MILES APART, AND DIFFERENCES OF LATITUDE AND LONGITUDE.

Lat.	Convergence Lks.	Convergence ' "	Difference of longitude per range: In arc	In time	Difference of latitude for— 1 mi.	1 Tp.
35	50.9	3 38	6 20.78	25.39		
36	52.7	3 46	6 25.53	25.70		
37	54.7	3 55	6 30.52	26.03	0.870	5.221
38	56.8	4 4	6 35.76	26.38		
39	58.8	4 13	6 41.27	26.75		
40	60.9	4 22	6 47.06	27.14		
41	63.1	4 31	6 53.15	27.54		
42	65.4	4 41	6 59.56	27.97	0.869	5.216
43	67.7	4 51	7 6.29	28.42		
44	70.1	5 1	7 13.39	28.89		
45	72.6	5 12	7 20.86	29.39		
46	75.2	5 23	7 28.74	29.92		
47	77.8	5 34	7 37.04	30.47	0.869	5.211
48	80.6	5 46	7 45.80	31.05		
49	83.5	5 59	7 55.05	31.67		
50	86.4	6 12	8 4.83	32.32		
51	89.6	6 25	8 15.17	33.03		
52	92.8	6 39	8 26.13	33.74	0.868	5.207
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
54	99.8	7 9	8 50.07	35.34		

Now from the standard field tables, again we go back to the standard field tables, and you know it is six miles apart again. We go over to this same table and we look at our 44 degrees again and this time we are going to look at the first line which is 70.1 links. And what that tells us is that in a township six miles by six miles.

If we survey around the exterior of that township and it is exactly six miles by six miles and all of those bearings are north south east west lines and we report true bearing on all of those lines, that township then will misclose by 70.1 links. It's telling us that in a township those meridians are converging by 70.1 links in that area in 36 square miles those are going to converge.

So we can work with that to calculate the convergency for any area of that township. So let's go back and look at our example and we will do this one at 48 degrees so linear convergency for a figure that has six square miles at 48 degrees. If we go to the table we will find that a township at 48 degrees has 80.6 links of linear convergency.

So it is a pretty simple matter to divide that by 36 square miles that gives us 2.2389 links per square mile and of course if we wanted to compute this factor in square feet or square chains, you know we could do it in anything just by what we divide it. So we've got 2.2389 links per square mile times six square miles gives us 13.43 links so at 48 degrees any figure, any closed figure that contains six square miles, doesn't matter the shape, doesn't matter, nothing else, just 48 degrees six square miles.

If we used true bearings all the way around it and we computed closure, its going to misclose by 13.43 links and what happens is if we go around clockwise, if we are calculating our misclosure clockwise, we are not going to have enough easting that's what is going to be short.

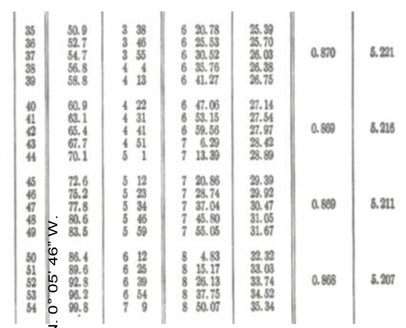
There is going to be more westing than there is easting because as we head north the meridians converge. So 13.43. And I just wanted to show you one other thing. If we look at our table again and we go to 48 degrees and we'll see that at 48 degrees there's 5.46 seconds of convergency in those meridians that are six miles apart.

RESTORATION OF LOST CORNERS – PART 2

Well, that means that if we started over there at the north east corner and we are now over at the north west corner or the south west corner, our bearing of this line six miles away is going to change by 5.46 seconds. That means that the bearing of the west boundary of this figure is 5 minutes and 46 seconds north west, it's not really north if we turn 90 degree angles. Well if we calculate, if we take the sign at 5 degrees 46 minutes times 80 degrees or 80 chains, we end up with 0.134 chains, 13.4 links, the same thing. It is a result of the convergence of the meridians is why we have that linear convergency.

Why it does not close and any time we are using true bearing, it is important that we take that into account. Now this has been a pretty, I guess a pretty quick review of what is going on with convergency and with curvature, with how it has an effect on what we do in the public land survey and how we need to apply it when we are doing our calculations for a single proportion when we are dealing with a range line or a township boundary, south boundary, north boundary of the township, any major east west line that has curvature in it, how we have to deal with it, how we can compute it, how we can account for that in our calculations, and put our corners when we are reestablishing corners back on the curve, how we can compute bearings and then how we need to deal with that linear convergency when we are all done to make sure that our figure really does close and our work is really accurate.

So I hope that this gets you along the way to understanding that. Of course in your material there is a good paper about curvature and convergency that I hope you read and then of course there is an exercise to kind of practice these skills and I hope that will end up being beneficial too. So hope that this is helpful we'll go on to the remainder of our discussion of the restoration of lost corners now.



35	50.9	3 38	6 20.78	25.39		
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50	86.4	6 12	8 4.83	32.32		
51	89.6	6 25	8 15.17	33.03		
52	92.8	6 39	8 26.13	33.74	0.868	5.207
53	96.2	6 54	8 37.75	34.52		
54	99.8	7 9	8 50.07	35.34		

N. 0° 05' 46" W.

Sin 0° 05' 46" x 80.00 = 0.134 chains (13.4 lks.)

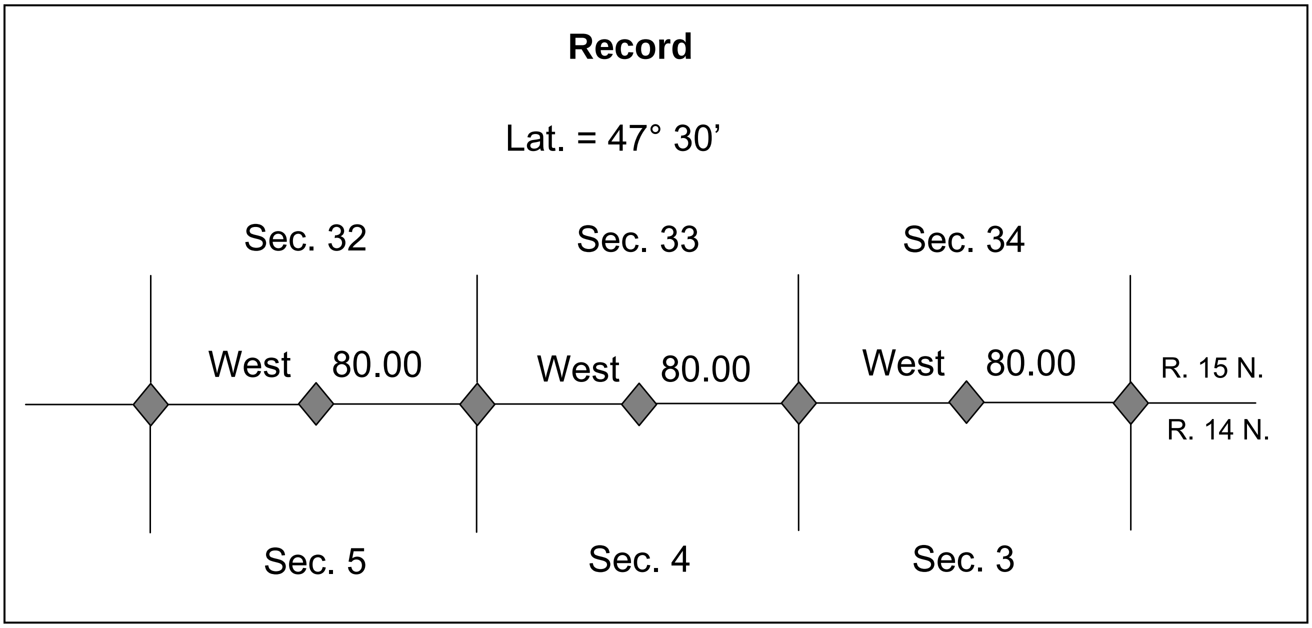
EXERCISE **Before moving on to the next topic, complete the "Single Proportion with Curvature Exercise" which can be found in the Exercise section at the end of this study guide.**

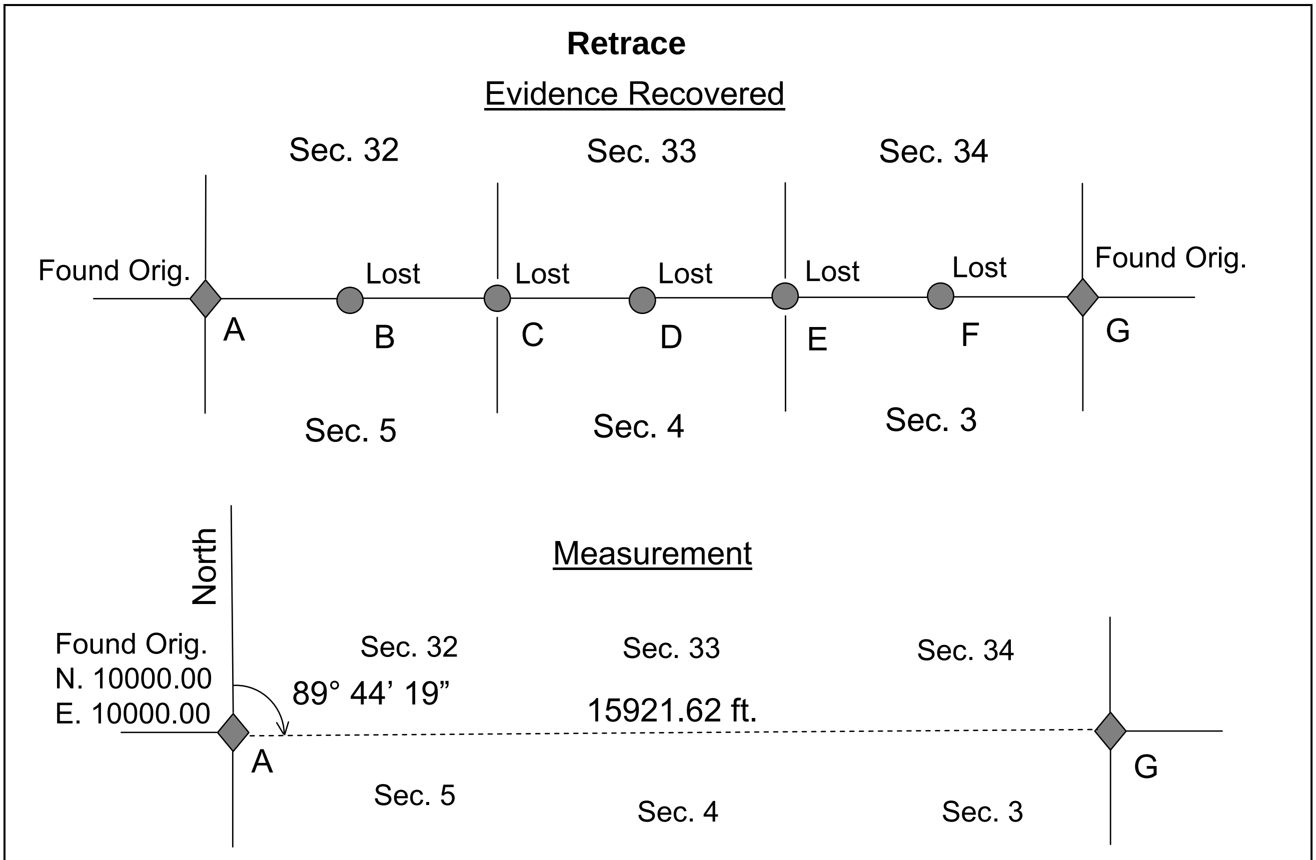
PROBLEM **Before moving on to the next topic, complete the Problem "Mean Bearing & Linear Convergency" which you can access from the course description page.**

HANDOUT **The paper on Curvature and Convergency can be found in the Handout section at the end of this study guide.**

EXERCISE

<u>Single Proportion with Curvature</u>





Calculate the proportionate position of the 5 lost corners.

<u>Calculate the True Mean Bearing of the Line</u>

Calculate forward bearing
Forward bearing at A (cor. of secs. 5, 6, 31 and 3)2: N. 0° 00' 00" E. + 89° 44' 19" = **N. 89° 44' 19" E.**

Calculate angular convergence:
Angular convergence for meridians 6 miles apart at Lat. 47° 30':
From Table 11: Convergence at 47° = 0° 05' 34"
Convergence at 48° = 0° 05' 46"
Convergence at 47° 30': (0° 05' 34" + 0° 05' 46") ÷ 2 = 0° 05' 40"
0° 05' 40" = 340"
340" ÷ 31680.00 ft.(6 miles) = 0.010732" per ft. of departure
0.010732" x 7960.73 ft. (1/2 departure of the line) = 0° 1' 25.44"
Mean bearing: N. 89° 44' 19" E. (forward bearing) **+** 0° 1' 25.44" (correction) = **N. 89° 45' 44.44" E.**
(The correction is applied clockwise because the bearing is easterly and we are going from forward bearing to true bearing)

Calculate departure of the line
Departure of the line: Sin 89° 44' 19 x 15921.62 = E. **15921.45 ft.**

Calculate the single proportion
The record calls for 6 equal 40 ch. segments, therefore: E. 15921.45 ÷ 6 = **E.2653.58 ft.**

Calculate the forward bearing from A to each of the lost corners

<u>Line A-B:</u>
E.2653.58 ÷ 2 = 1326.79 ft. (1/2 the departure of line A-B)
E.1326.79 x 0.010732" (angular convergence per ft. of departure) = 0° 00' 14.24" (angular convergence)
N. 89° 45' 44.44" E. (mean bearing line A-G) - 0° 00' 14.24"(angular convergence) = **N. 89° 45' 30.24" E.**
(We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-B: 2653.58 ft. (departure of A-B) ÷ sin 89° 45' 30.24" = **2653.60 ft.**

At Pt. A the forward bearing and distance to Pt. B on the curve is:
N. 89° 45' 30.24" E., 2653.60 ft

Coordinates of the proportioned point: N.10011.19, E.12653.58

<u>Line A-C:</u>
E.5307.16 ÷ 2 = 2653.58 ft. (1/2 the departure of line A-C)
E.2653.58 x 0.010732" (angular convergence per ft. of departure) = 0° 00' 28.48" (angular convergence)
N. 89° 45' 44.44" E. (mean bearing line A-G) - 0° 00' 28.48"(angular convergence) = **N. 89° 45' 15.96" E.**

(We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-C: 5307.16 ft. (departure of A-C) ÷ sin 89° 45' 15.96" = **5307.21 ft.**

At Pt. A the forward bearing and distance to Pt. C on the curve is:
N. 89° 45' 15.96" E., 5307.21 ft

Coordinates of the proportioned point: N.10022.75, E.15307.16

<u>Line A-D:</u>
E.7960.74 ÷ 2 = 3980.37 ft. (1/2 the departure of line A-D)
E.3980.37 x 0.010732" (angular convergence per ft. of departure) = 0° 00' 42.72" (angular convergence)
N. 89° 45' 44.44" E. (mean bearing line A-G) - 0° 00' 42.72"(angular convergence) = **N. 89° 45' 01.72" E.**
(We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-D: 7960.74 ft. (departure of A-D) ÷ sin 89° 45' 01.72" = **7960.81 ft.**

At Pt. A, the forward bearing and distance to Pt. D on the curve is:
N. 89° 45' 01.72" E., 7960.81 ft.

Coordinates of the proportioned point: N.10034.67, E.17960.74

<u>Line A-E:</u>
E.10614.32 ÷ 2 = 5307.16 ft. (1/2 the departure of line A-E)
E.5307.16 x 0.010732" (angular convergence per ft. of departure) = 0° 00' 56.96" (angular convergence)
N. 89° 45' 44.44" E. (mean bearing line A-G) - 0° 00' 56.96"(angular convergence) = **N. 89° 44' 47.48" E.**
(We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-E: 10614.32 ft. (departure of A-E) ÷ sin 89° 44' 47.48" = **10614.42 ft.**

At Pt. A, the forward bearing and distance to Pt. E on the curve is:
N. 89° 44' 47.48" E., 10614.42 ft.

Coordinates of the proportioned point: N.10046.96, E.20614.32

<u>Line A-F:</u>
E.13267.90 ÷ 2 = 6633.95 ft. (1/2 the departure of line A-F)
E.6633.95 x 0.010732" (angular convergence per ft. of departure) = 0° 01' 11.20" (angular convergence)
N. 89° 45' 44.44" E. (mean bearing line A-G) - 0° 01' 11.2"(angular convergence) = **N. 89° 44' 33.24" E.**
(We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)

Distance of line A-F: 13267.90 ft. (departure of A-F) ÷ sin 89° 44' 33.24" = **13268.03 ft.**

At Pt. A, the forward bearing and distance to Pt. F on the curve is:
N. 89° 44' 33.24" E., 13268.03 ft.

Coordinates of the proportioned point: N.10059.61, E.23267.90

<u>Line A-G:</u>
E.15921.48 ÷ 2 = 7960.74 ft. (1/2 the departure of line A-G)
E.7960.74 x 0.010732" (angular convergence per ft. of departure) = 0° 01' 25.43" (angular convergence)
N. 89° 45' 44.44" E. (mean bearing line A-G) - 0° 01' 25.43"(angular convergence) = **N. 89° 44' 19.01" E.**
(We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-G: 15921.48 ft. (departure of A-G) ÷ sin 89° 44' 19.01" = **15921.65 ft.**

At Pt. A, the forward bearing and distance to Pt. F on the curve is:
N. 89° 44' 19.01" E., 15921.65 ft *(notice this agrees with the measured forward bearing and distance of line A-G. The minor difference in distance is the result of rounding)*

Coordinates of the proportioned point: N.10072.64, E.25921.48


HANDOUT

<u>Discussion of Convergency</u>

Surveys may be divided into two classes: plane and geodetic. The first treats the surface of the earth as a plane; the second treats it as the surface of a spheroid. Plane surveying is suitable for small areas because the effect of curvature of the surface of the earth is not appreciable over short distances. Over longer distances, the effect of curvature becomes significant.

One way in which the effect of curvature manifests itself is in the convergence of the meridians. This convergence may be observed on a globe of the world; all meridians meet at the poles. Since they are great circles of the earth, they may be laid out on the ground as converging straight lines. Thus in northern latitudes, the distance between two meridians becomes less as one proceeds northward.

Parallels of latitude (except the equator) are not great circles of the earth. They do not converge and they may not be laid out as straight lines, either on the ground or on a map. They must be laid out as circular curves. The degree of curvature of these parallels becomes greater and greater as one proceeds north in northern latitudes; but everywhere the curvature of a parallel must be such that it crosses every meridian at right angles. The parallel of latitude are everywhere true east-west lines; and the meridians true north-south lines.

The law requires that the north-south township boundaries shall follow true meridians and that the east-west township boundaries must cross the meridians at right angles, i.e. they must follow parallels of latitude. The problem then resolves itself into one of devising a practical method of laying out these curved parallels of latitude on the ground with sufficient accuracy to satisfy the requirement of the law and the demands of good surveying practice. Three methods are commonly employed: <u>The Tangent Method</u>, <u>The Secant Method</u> and <u>The Chord Method</u>.

<u>The Tangent Method</u>: This method of laying out the latitude curve on the ground consists of : (1) orienting the instrument in the true meridian; (2) turning a 90-degree angle right or left as the case may be, and projecting a straight line for six miles; (3) measuring half mile intervals along this straight line and at the end of each half mile measure an offset north and establishing a point on the latitude curve. The straight line, being 90 degrees to the meridian at the point of beginning, is tangent to the parallel of latitude at that point and has a true east-west direction. East of that point will have an increasingly southeasterly direction, and west of that point an increasingly southwesterly direction. Figure 14, page 154 of the manual illustrates the establishment of a true parallel east from the point of beginning. Offsets from the tangent through its six mile length are tabulated in the <u>Standard Field Tables</u>, Tables 12 & 13 page 200 & 201, for latitudes between 25 and 70 degrees. Intermediate offsets may be computed on the assumption that the lengths of the offsets vary as the squares of their distances from the point of beginning. Actually a series of points thus determined would define a parabola, but results thus determined are well within the expected limits of accuracy of the survey.

The Secant Method: This method is a modification of the tangent method, designed to keep all offsets relatively short. As shown in Figure 15, page 156 of the manual the secant is a straight line six miles long. It cuts the latitude curve at the first and fifth miles. It is parallel to an imaginary line drawn tangent to the latitude curve at the third mile. Its bearing is southwest or northeast depending upon the direction of projection. East of the third mile its bearing is southeast or northwest, depending upon the direction of projection. Azimuths of the secant throughout its six mile length and offsets therefrom to the latitude curve at half mile intervals are given in the Standard Field Tables, Tables 14 & 15, pages 202-203, for latitudes from 25 to 70 degrees.

To lay out the latitude curve by the secant method: From a known point on the latitude curve at mile zero, measure the tabulated offset south to the zero end of the secant. With the instrument on the point thus determined and oriented in the meridian, turn off the proper angle to establish the tabulated bearing of the secant at that point. Project the secant as a straight line for six miles. As measurements are completed to each corner, measure the proper offset north or south and establish the corner. Observe that offsets are zero at the first and fifth miles.

The Chord Method: This method is a modification of the secant method. The offset from the chord are increased by the amount of the offset from the secant at mile zero. The advantages of this method are that all offsets are south (in north latitudes) and the line is started at the corner (mile zero point) without an offset.

To lay out the latitude curve by the cord method: From a known point on the latitude curve at mile zero, orient the instrument to the meridian and turn off the proper angle to establish the tabulated bearing of the secant at that point. Project the chord as a straight line for six miles. As measurements are completed to each corner, measure the proper offset south and establish the corner.

CONVERGENCE OF THE MERIDIANS

An idea of the amount of angular convergence of the meridians may be seen from an examination of figures 14 and 15 of the manual. The values of angular convergence are shown in the Standard Field Tables, Table 11, page 199, for meridians six miles apart, in latitudes from 25 to 70 degrees. If a straight line crosses two meridians the difference between the corresponding angles formed by the line and the meridians represents the angular convergence of the meridians between the two points depends upon their difference in longitude (i.e., the departure of the line joining them) and also upon the mean latitude of the two points.

In adjusting the courses of a traverse so as to take angular convergence into account, the total angular convergence is distributed so that the amount ascribed to each course bears to the total departure of the traverse. The Standard Field Tables show the amount of linear convergence of two meridians six miles long and six miles apart, e.g., the convergence of the meridional boundaries of a township, which is to say the amount by which the north boundary of a township is shorter than its south boundary. In using the table the mean latitude between the north and south boundaries should be employed.

In any given mean latitude the amount of linear convergence of two meridians depends upon their length and the distance between them. From this is follows that linear convergence is a function of the area, i.e., length times breadth. Thus in dealing with an irregularly shaped survey the effect of linear convergence may be regarded as a function of the area. Its amount will be in the same proportion as the area of the figure is to the tabulated values for 36 square miles (township) in the Standard Field Tables, Table 11, page 199.

Rhumb Lines and Loxodromes

As noted above, straight lines on the earth's surface do not have a constant bearing throughout their lengths. (Exceptions are the meridians and the equator). Conversely, lines of constant bearing (excepting the meridians and equator) are curved lines. Lines of constant bearing are called loxodromes or rhumb lines. The curvature of a loxodrome may be circular, as in the case of latitudinal curves; or it may be in the form of a spiral. All lines of constant bearing,, excepting those running in the cardinal directions have the form of spirals.

Although it is possible to calculate the form of such spirals it is often regarded as impractical to do so. Frequently, sufficient accuracy may be attained by assuming that a straight line, having a mean bearing equivalent to that of the required loxodrome will serve. The mean bearing of such a line is taken to be the average of the forward bearings at its two ends, i.e., the average of its forward bearing and the 180 degree opposite of its back bearing. The foregoing assumption may lead to intolerable error, however, in the case of very long lines. A concept of the validity of the assumption may be gained from figure 15 of the manual showing the secant method of laying off the true parallel. The bearing "East" at the center is the mean between the bearings at the ends.

An acceptable method of laying out a loxodrome upon the ground by foresights and backsights with a surveyor's transit consists of calculating the angular convergence between the ends of the line and turning proportional deflection angles along the line, thus producing a series of chords which closely approximate a line of constant bearing.

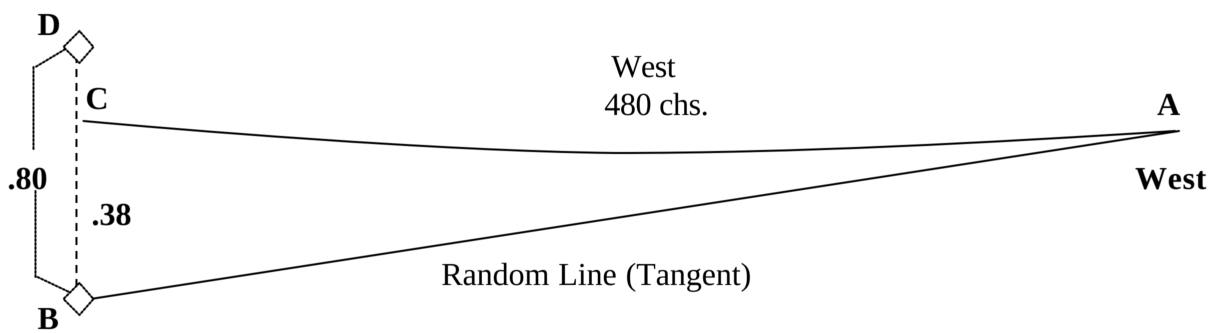
CONVERGENCY
AND
RESURVEY

Example I:

You have been assigned to resurvey the S. boundary of Tp._______, R. _____, latitude 46° N. Your instructions call for the recovery or reestablishment of all lost corners along the line. The record bearing and distance for this line is West, 480 chs. With all intermediate corners established at 40 chain intervals along a <u>true</u> parallel of latitude.

FIGURE 1

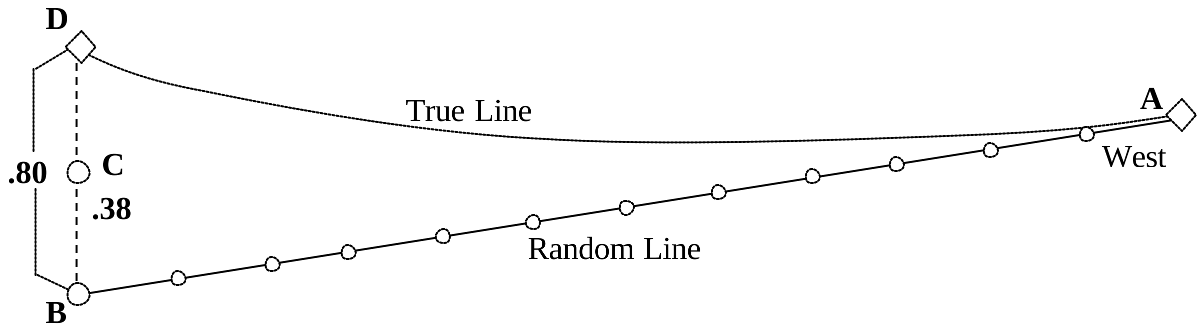
Lat. 46° N.



In retracing the line you determined the meridian at the original corner 'A,' Figure 1 and turned west, extending the line and placing temporary corner stakes at ½ mile intervals. When you turned west your intent was to retrace the record by turning the tangent to the record line A-C, Figure 1. The search for corners resulted in the recovery of only the original township corner 'D' at a distance of 480 chs. (Record Dist.) and 0.80 ch. North of the temp. 'B.'

According to table 13, pg. 201, Standard Field Tables, the offset from the tangent to the parallel at lat. 46° N. is 38 lks. (0.38 chs.) If this corner was in accord with the record it would have been found 0.38 chs. North of your 6 mile (480 ch.) temp. The actual measurement from your temp to the found corner was 0.80 chs. Figure 2 shows the new configuration of the line between the two found corners.

FIGURE 2

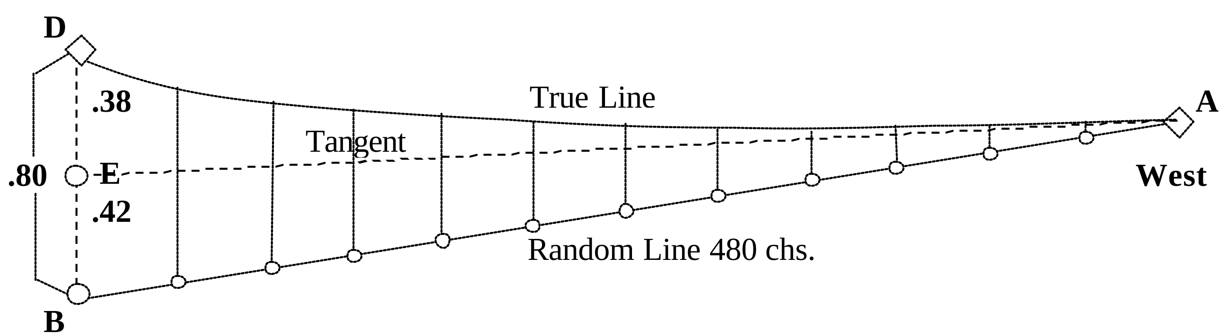


The <u>curved</u> line A-D now becomes the true line and the line A-B becomes a random line and <u>is not tangent</u> to the curved line A-D. Since our random line is not tangent to the true line the problem becomes one of determining the moves from the temps to the true line, keeping in mind that the true line is actually curving away from its own tangent and the random line.

The solution to the problem is simply one of solving a series of similar plane triangles and applying offsets from the tangent to the true curved line.

Figure 3 shows the construction of a tangent A-E to the true line and the forming of the plane triangle A E B.

FIGURE 3



The offset from the tangent D-E is taken from the tables for lat. 46° N. Points A and E are connected by a straight line thus forming a tangent to the true line A-D.

The offset from the tangent D-E (.38) is now subtracted from measurement B-D (.80) to obtain the distance B-E (.42) thus forming the base of triangle A E B.

The distances from the random line temps can be determined from the ratio BE : AB, (Figure 3). These results are then added to the tabulated offsets from tangent for lat. 46° N. (Standard Field Tables, Table 13)

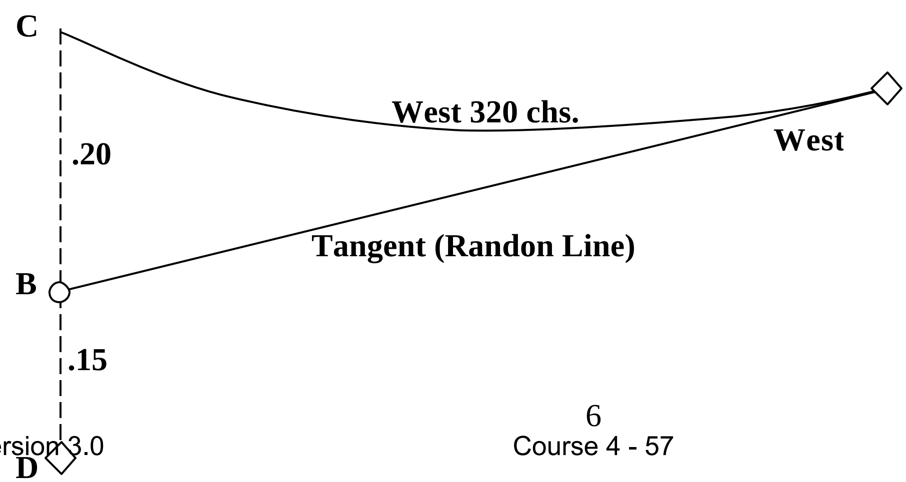
$$\frac{42}{480} = .000875 \quad \text{(Constant Multiplier)}$$

Temp.	Dist.		Constant		Proportion		O.S. From Tan. Table 13, 46°		Move from Temp.
1	40	x	.000875	=	.035	+	.00	=	.035 N
2	80		"		.070		.01		.080
3	120		"		.105		.02		.125
4	160		"		.140		.04		.180
5	200		"		.175		.07		.240
6	240		"		.210		.09		.300
7	280		"		.245		.13		.375
8	320		"		.280		.17		.450
9	360		"		.315		.21		.525
10	400		"		.350		.26		.610
11	440		"		.385		.32		.705
B	480		"		*.420		.38		*.800
					*check				*check

Example II:

Your instructions call for the recovery or reestablishment of all corners on the west four miles of the N. Bdy. Of Tp. _____, R._____, in latitude 51° N. The record is West, 320 chs. with all intermediate corners established at 40 chain intervals along a true parallel of latitude.

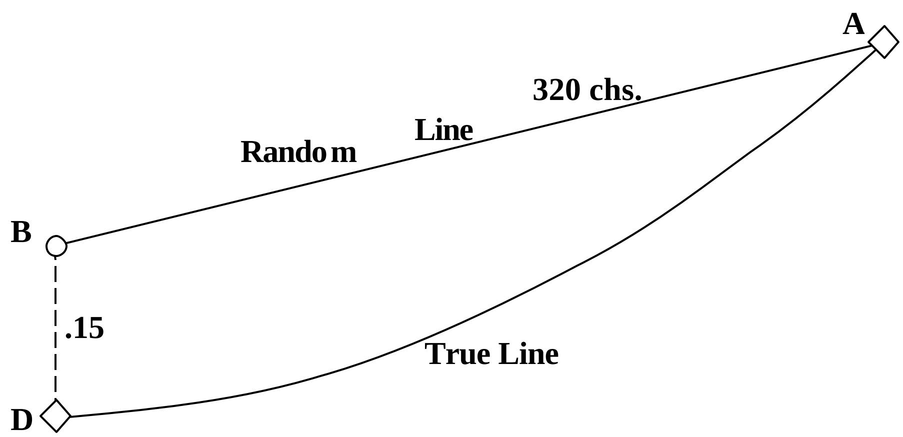
Figure 4



In retracing the line you determined the meridian at the original corner ‘A,’ Fig. 4 and turned West, extending the line and placing temporary corner stakes at ½ mile intervals. When you turned West your intention was to retrace the record by running the tangent to the record line A-C. The search for corners resulted in the recovery of only the original township corner ‘D’ at a distance of 320 chs. (Record Dist.) and 0.15 chs. South of the temp. ‘B.’

According to table 13, page 201, Standard Field Tables the offset from the tangent to the parallel for 4 miles, in lat. 51° is 20 lks. (0.20 chs.) If this corner was in accord with the record it would have been found 0.20 chs. North of temp. B. The actual measurement from the temp. to the corner was 0.15 chs. South. Figure 5 shows the new configuration of the line between the two found corners.

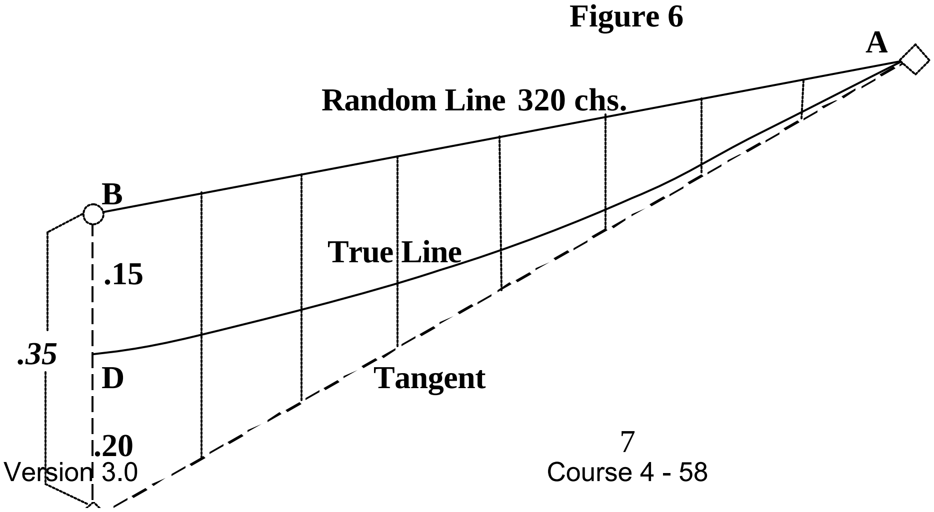
Figure 5



The curved line A-D now becomes the true line and the line A-B becomes a random line and is apparently not tangent to curved line A-D. The problem of determining the moves from the random line temps to the true line is basically the same as that in Example I.

Figure 6 shows the construction of the tangent A-E and the formation of triangle A E B.

Figure 6



As in Example I, we can now compute the values of the lines from the temps to the tangent and by subtracting the offsets from the tangent for 51° we have the moves from the random line temps to the true line; in this case all moves are south.

Then:

$$\frac{B\,E}{A\,E} = \frac{.35}{320} = .001094 \text{ (Constant Multiplier)}$$

Temp.	Dist.		Constant		Proportion		O.S. From Tan. Table 13, 51°		Move From Temp.
1	40	x	.001094	=	.044	-	.00	=	.044
2	80		"		.087		.01		.077
3	120		"		.131		.03		.101
4	160		"		.175		.05		.125
5	200		"		.219		.08		.139
6	240		"		.262		.11		.152
7	280		"		.306		.15		.156
B	320		"		*.350		.20		*.150
					*check				*check

In the foregoing examples we have retraced the record line by running the tangent to the true parallel of latitude. After locating the controlling corner the line that was run as a tangent to record the true line loses its value as a tangent to the true line because the true line has assumed a position other than that of the record. Therefore the line run as a tangent becomes a random line. From this point it can be seen that the random line does not need to be tangent to the record line. It can be any line run in the general direction of the controlling corner. It can be the chord or secant of the record – or either of these. In any case the calculations can be performed in the same manner as shown in the examples.

The method we have discussed is a close approximation. The results obtained are, for all practical purposes, well within the required accuracy for land surveys as long as the survey is not extended beyond the limits of the values tabulated in the Standard Field Tables.

OFFSET FROM TANGENT

OFFSET IN LINKS =

$$= 1.01 \times \text{TAN } F \times \text{MILES}^2 \qquad \text{(SIN BRG.)}$$

$$= 0.000158 \times \text{TAN F} \times \text{CHAINS}^2 \quad (\text{SIN BRG.})$$

F = MEAN LATITUDE

EXAMPLES:

LINE = N. 80° 00’ E., 640 CHS. (8 MILES)

F = 41° 30’ N.

$$\text{OFFSET} = 1.01 \times .88473 \times (8)^2 \times .98481 = 56.32 \text{ LKS.}$$

<u>OR</u>

$$\text{OFFSET} = .000158 \times .88473 \times (640)^2 \times .98481 = 56.39 \text{ LKS.}$$

OFFSETS FOR TANGENTS
FACTORS FOR OFFSETS IN LINKS

LAT.	"C" Factor	LAT.	"C" Factor	LAT.	"C" Factor
30º 00'	0.90971	45º 00'	1.57433	60 º 00'	2.72451
30 30	0.92811	45 30	1.60200	60 30	2.78018
31 00	0.94670	46 00	1.63013	61 00	2.83743
31 30	0.96549	46 30	1.65885	61 30	2.89687
32 00	0.98448	47 00	1.68806	62 00	2.95807
32 30	1.00368	47 30	1.71783	62 30	3.02131
33 00	1.02309	48 00	1.74816	63 00	3.08671
33 30	1.04272	48 30	1.77909	63 30	3.15439
34 00	1.06257	49 00	1.81063	64 00	3.22447
34 30	1.08266	49 30	1.84281	64 30	3.29713
35 00	1.10300	50 00	1.87566	65 00	3.37247
35 30	1.12358	50 30	1.90919	65 30	3.45071
36 00	1.14442	51 00	1.94345	66 00	3.53199
36 30	1.16552	51 30	1.97844	66 30	3.61652
37 00	1.18690	52 00	2.01422	67 00	3.70452
37 30	1.20856	52 30	2.05080	67 30	3.79619
38 00	1.23050	53 00	2.08824	68 00	3.89185
38 30	1.25276	53 30	2.12652	68 30	3.99171
39 00	1.27532	54 00	2.16574	69 00	4.09610
39 30	1.29820	54 30	2.20591	69 30	4.20535
40 00	1.32141	55 00	2.24707	70 00	4.31981
40 30	1.34496	55 30	2.28927	70 30	4.43992
41 00	1.36887	56 00	2.33256	71 00	4.56616
41 30	1.39314	56 30	2.37698	71 30	4.69969
42 00	1.41778	57 00	2.42258	72 00	4.83864
42 30	1.44282	57 30	2.46943	72 30	4.98620
43 00	1.46826	58 00	2.51758	73 00	5.14217
43 30	1.49411	58 30	2.56709	73 30	5.30727
44 00	1.52040	59 00	2.61804	74 00	5.48246
44 30	1.54713	59 30	2.67048	74 30	5.66859
45 00	1.57433	60 00	2.72451	75 00	5.86687

C= "C" factor from above table. D=Distance in chains, East or West

M = Move, in links, from the Tangent to the Parallel

$$M = (D \backslash 100)^2 C$$

CONVERGENCY OF MERIDIANS

NOTATION:

C = CONVERGENCY IN SECONDS

F = MEAN LATITUDE

C = .6501 x TAN F x CHAINS x SINE BRG.

C = 52.008 x TAN F x MILES x SINE BRG.

EXAMPLE:

LINE=N. 80º 00' E., 640 CHS. OR 8 MILES

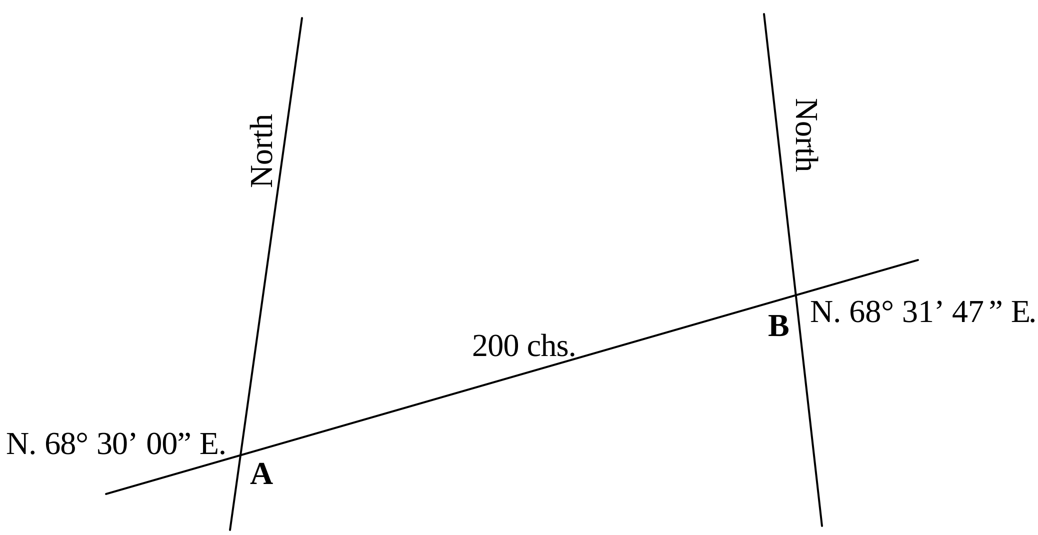
F = 41º 31' N.

C = .6501 x .88473 x 640 x .98481 = 362.51"

OR

C = 52.008 x .88473 x 8 x .98481 = 362.51"

ANGULAR CONVERGENCY
HOW TO APPLY IT



Mean Latitude
41 degrees and 30 min.

Example: I have run a straight line (AB) on a beginning bearing of N. 68º 30' 00"E., for a distance of 200 chs. I have computed the angular convergency to be 0° 01' 47". What is the bearing of the line at B?

Rule: If the latitude of the line is increasing with its direction, add the correction for convergency.
If the latitude of the line is decreasing with its direction, subtract the correction for convergency.

Therefore: The beginning bearing is N. 68º 30' 00" E. Since the latitude of the course is increasing, I add the convergency: 68º 01' 00" + 0° 01' 47" and the bearing of the line at point B becomes N. 68º 31' 47" E.

LINEAR CONVERGENCY

The linear Convergency of a township or of meridians is an amount expressed in linear measurement; e.g., miles, chains, links; of the effect due to the meridians converging at the poles.

Bluntly: We are doing plane surveying on a round surface and must make adjustments accordingly.

LINEAR CONVERGENCY OF MERIDIANS

Linear convergency of survey figures may be computed as follows:

(A) Using BLM STANDARD FIELD TABLES in Table 11 with argument as the approximate mean latitude (see example 1).

(B) By the formula

$$dm? = \frac{m?\ mF}{a} \qquad \tan f\ v\overline{(1-e^2\ Sin^2\ F)}$$

(see example 2)

(C) For fractional parts of a township. (see example 3)
The 1947 Manual, Section 129 states:

"Simple interpolatioin may be made for any intermediate latitude, and the amount of the convergency for a fractional township or other figure may be taken in proportion to the tabulated convergency as the fractional area is to 36 square miles."

(D) For irregular figures. (see example 4)
The 1947 Manuel, Section 129 also states:

"The correction for convergency in any closed figure is proportional to the area and may be computed from an equivalent rectangular area."

(E) For a single course. (see example 5)

EXAMPLE I

Linear convergency of a township (6 miles long by 6 miles wide) at mean latitude 48º 00' N., can be looked up directly in the STANDARD FIELD TABLES in Table 11.
For this example the linear amount of convergency is 80.6 links.
With the same data at mean latitude 26º 30' N., a straight line interpolation is made between mean latitudes 26º 00' and 27º 00' to obtain 36.2 links as the amount of convergency in the township.
With the same data at mean latitude 42º 00' N., the convergency would be 65.4 links.

Note: Compare with example 2.

EXAMPLE 2

The linear convergency of meridians of a township or line may be found by the formula in the STANDARD FIELD TABLES (page 224):

$$dm? \quad : \quad \frac{m?\ mF}{a} \qquad \tan f\ v\overline{(1-e^2\ Sin^2F)}$$

dm? : Linear amount of convergency

m? : Measurement along the parallel

m F : Measurement along the meridian

a : Equitorial radius of the earth 317064 chains

e : Factor of eccentricity log e 8.9152515-10

e^2 : e in natural numbers squared 0.006768658

F : Approximate mean latitude

e was given in log form because previous calculators were not made to handle the squaring and multiplication under the radical with proper precision.

Given : mean latitude of 42º 00' N.

$$dm? = \frac{m?\ mF}{a} \quad \overline{\tan f\ v(1\text{-}e^2}\ Sin^2F)$$

$$dm? = \frac{(480chs)\ (480chs)}{317064\ chs} \quad \tan 42^o\ v\overline{(1\text{-}e^2\ Sin^2 42^o}) $$

$$dm? = (.72667chs)\ (.90040)\ (v\overline{1\text{-}e^2 \sin^2 42^o})$$

$$dm? = (.72667chs)\ (.90040)\ (v\overline{1\text{-}(0.006768658)\ (0.4477357686)}\ )$$

$$dm? = (.72667chs)\ (.90040)\ (v\overline{.99696943})$$

$$dm? = (.72667chs)\ (.90040)\ (.99848)$$

dm? = .653 chs.

EXAMPLE 3

This method is very appropriate for finding the linear convergency for any portion of a township: e.g., one section, two sections, etc., or any closed figure at the same approximate mean latitude.

METHOD: $\dfrac{\text{conv. (from Table 11)}}{\text{36 square miles}} = \dfrac{\text{Conv.}}{\text{“Any Area”}}$

(1) Given: mean latitude 49º 00' N.

Find: Conv. For 2 sections

$$\frac{\text{83.5 links}}{\text{36 Square miles}} = \frac{\text{Conv.}}{\text{2 square miles}}$$

Conv. = 04.6 links

(2) Given: mean latitude 34º 30' N.

Find: Conv. For 8.4 square miles

$$\frac{50.0 \text{ links}}{36 \text{ square miles}} = \frac{\text{Conv.}}{8.4 \text{ square miles}}$$

Conv. = 11.7 links

EXAMPLE 4

The area of any closed figure can be described by an equivalent rectangular area. e.g., an area of 9.7 square miles can be reduced to an equivalent rectangular area of a rectangle 4.85 miles by 2 miles or 2.425 miles by 4 miles, etc.

Formula: Conv. = .0202 d 1 tan F

(d) (1) is length times width in miles which yields area

(1) Given: mean latitude 32º 00' N
Area of figure = 22.8 sq. miles

Find: convergency in chains

Conv. = (.0202) (2 miles) (11.4 miles) (tan 32º)

Conv. = .287 chains

Note: This formula yields answer in chains. May be used for townships or any portion of a township.

(2) Given: mean latitude 34º 30' N.

Find: Conv. For 8.4 square miles

Conv. = (.0202) (2 miles) (4.2 miles) (tan 34º 30')

Conv. = .117 chains

Note: Compare answer to example 3 part 2

RESURVEY METHODS
CURVATURE
(SECONDARY METHOD)

An alternate method of computing the curvature and incorporating its effect into corner moves along a standard parallel or township line is by the mean bearing method.

In this method the amount of curvature for the latitude of the line is taken from the standard field tables and the proper amount applied to the bearing of the portion of the random line concerned. The latitudes and departures of the traverse are computed and accumulated and the mean bearing of the true line is determined. The true line proportional latitudes and departures are then computed and accumulated. The difference between the true line accumulated data and the random accumulated data for each increment of the survey line is the move. The move then includes the adjustment for curvature.