UNIT 3

BASIC MEASUREMENTS

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Introduction

Philip Kissum in his book, *Surveying Practice,* defines surveying as "the art, science and technology of making such measurements as are necessary to determine the relative position of points above, on or beneath the surface of the earth, or to establish such points in a specified position." The key terms in the above definition are art, science, technology and measurements. The science and technology of surveying are easy to recognize. The science includes the laws of mathematics and physics that surveyors use every day while making measurements. Technology includes the instrumentation, calculation devices, and mapping technologies (CADD) that have become necessities in current survey practice. The art of surveying is a little harder to define, but includes the use of judgment, gained through experience, that allows surveyors to choose the technologies and procedures to do a project correctly and efficiently.

Measurements are the cornerstones upon which the surveyor builds experience. Without a thorough understanding of the basic survey measurements, a surveyor cannot expect to move onto the more complicated technical issues nor onto the professional issues. This unit of the training program will deal with the basic survey measurements of distance, direction and elevation. The unit will also concentrate on measurement analysis which includes the study of errors and how they affect measurements.

Performance Expected on the Exams

Given sample measurements, determine the most probable value, the standard error and the standard error of the mean.

Given the standard error for certain measurements, equipment, and procedures, predict the outcome of a survey using these measurements.

Explain the difference between slope and horizontal distance. Calculate horizontal distances from slope distances and vertical angles or differences in elevation.

Explain the effects of systematic errors in distance measurements (i.e., temperature correction, sag correction, etc.) and be able to calculate their value.

Explain the fundamental principles of taping and Electronic Distance Measurement (EDM) operation.

Explain the basic methods and procedures for using a theodolite.

Explain the differences between angles, bearings and azimuths and be able to convert between them.

Key Terms

Measurements

Systematic errors

Mistakes

Accuracy

Mean

Standard error

Slope distance

Vertical angle

Taping

Elevation

Direction

Bearings

Magnetic azimuths

Double centering

Closing the horizon

Significant figures

Random errors

Precision

Probability

Residuals

Error of the mean

Horizontal distance

Vertical distance

Electronic distance measurement (EDM)

Leveling

Angles

Azimuths

Theodolite

Wiggling in

Video Presentation Outline

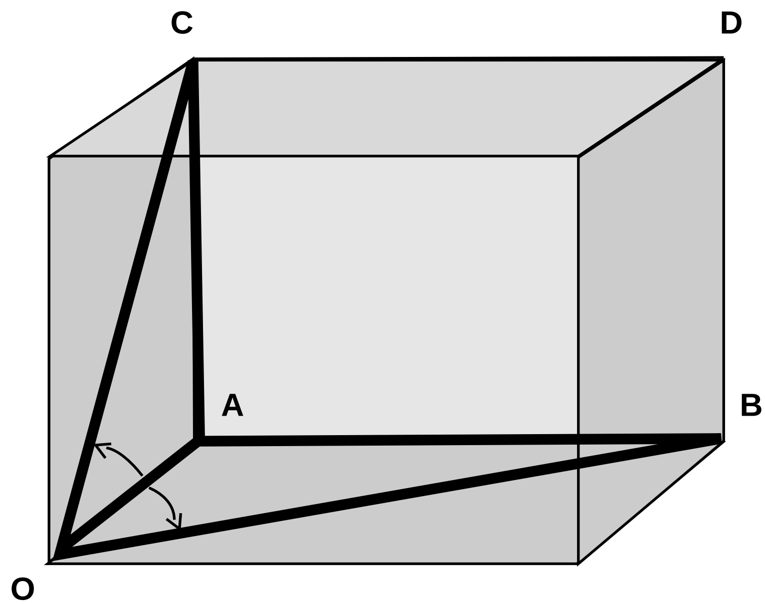


Figure 3-1. Measurements in three dimensions.

Types of Measurements

- Horizontal angles
- Horizontal distances
- Vertical angles
- Vertical distances
- Slope distances

Units of Measurements

- Length
- Angle
- Area
- Volume

Significant Figures

Numerical Value	Significant Figures
49.00	2
1600.00	2 or 4
.1284	4
0.21	2
00.000213	3
129.85	5
11.00	2 or 4
10,000.0001	9
5,280.00 ft/mile	infinite

- Number of significant figures
- Significant figures in addition and subtraction
- Significant figures in multiplication and division
- Problems relating to significant figures in surveying

Significant Figure Problem

Answer the following problems with the correct number of significant figures.

- Sum of 19.27, 0.0000345, 121 and 8.6
- Product of 28953.844 and 1.34
- Sum of 0.025, 1456.3, 18.466 and 438.6
- Quotient of 9430.445 divided by 9.45

Direct and Indirect Measurements

- Direct measurements
- Indirect measurements

Error Theory

- Errors in measurements:
  1. Error(E) = Measured Value - True Value

- Sources of error in making measurements:
  1. Natural error
  2. Instrumental error
  3. Personal error

- Types of errors:
  1. Systematic errors
  2. Random errors

- Precision and accuracy:

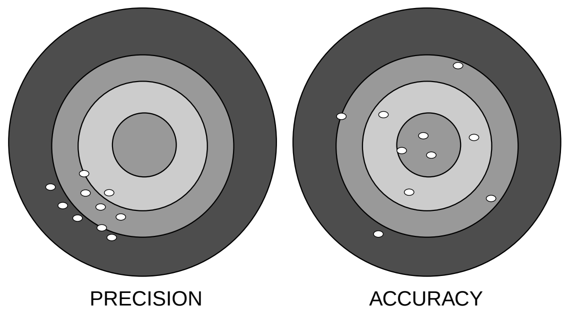


Figure 3-2. *Accuracy and Precision.*

- Eliminating mistakes and systematic errors
- Probability:

Measurement	r	r^2
1. 56° 23' 45"		
2. 56° 23' 46"		
3. 56° 23' 45"		
4. 56° 23' 43"		
5. 56° 23' 44"		
6. 56° 23' 45"		
7. 56° 23' 46"		
8. 56° 23' 45"		
9. 56° 23' 44"		
10. 56° 23' 46"		
11. 56° 23' 47"		
12. 56° 23' 44"		
Σ		

Most Probable Value

- Mean

Measures of Precision

- Residual(r) = measurement - most probable value
- Histogram
- Standard deviation (standard error)

  $= \pm\sqrt{\frac{\sum r^2}{n-1}}$

- 50%, 90% and 95% errors:

  Probable error (E_{50}) = ±0.6745 (SD)

  90% Error (E_{90}) = ±1.6449 (SD)

  95% Error (E_{95}) = ±1.9599 (SD)

  Where:

  SD = Standard deviation

Measures of Accuracy

- Standard error of the mean:

  $E_m = \pm\frac{SD}{\sqrt{n}}$

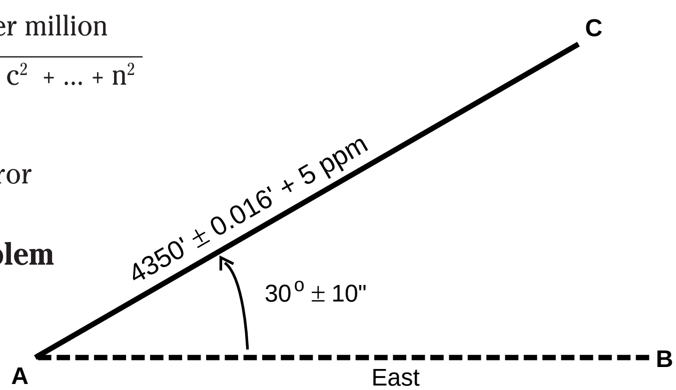
Error Propagation

- $E_{series} = \pm E\sqrt{n}$
- $E_{edm} = \pm E$ + Parts per million
- $E_{sum} = \pm\sqrt{a^2 + b^2 + c^2 + \ldots + n^2}$

  Where:

  E = Constant error

Point Location Problem



Determine the error in the position of Point C given the data as shown.

Estimated Traverse Closure Problem

Determine the estimated and actual closure of the traverse data shown below:

Side	Azimuth	Error	Distance	Error
A-B	90° 00' 00"	± 27"	1152.66'	±0.15'
B-C	184° 15' 16"	± 44"	501.92'	±0.14'
C-D	274° 49' 14"	± 18"	1192.10'	±0.10'
D-A	10° 15' 30"	± 58"	406.55'	±0.20'

Classifications of Traverse Accuracy Standards

	2nd Order (Modified)	3rd Order
Maximum number of courses between checks for azimuth	15	25
Azimuth closure not to exceed*	$10''\sqrt{N}$ or $3''$ per station	$30''\sqrt{N}$ or $8''$ per station
Position closure (after azimuth) adjustment not to exceed*	$1.67'\sqrt{M}$ or 1:10,000	$3.34'\sqrt{M}$ or 1:5000
Distance Measurement accurate within	1:15,000	1:7,500
Minimum distance to be measured with EDMs	0.1 mile	0.05 mile
Minimum number of Angle Observations:		
a) One-second theodolite	4 pos.	2 pos.
b) One-minute or 20-second theodolite	1 set of 6D, 6R	1 set of 2D, 2R

Notes:
1. N is the number of stations for carrying azimuth
2. M is the distance in miles

* *The expressions for closing errors in traverse surveys are given in two forms. The formula which gives the smaller permissible closure should be used.*

Table 3-1. *Table 4-04-B, "Caltrans Surveys Manual."*

Adjustments

- Traditional adjustments (proportional/linear adjustments)
- Least squares

Distance Measurements

Methods of Measuring Horizontal Distances

- Pacing
- Odometer
- Optical rangefinders
- Tacheometry (stadia)
- Subtense bars
- Taping
- EDM

Distance Measurements by Taping

- Basic procedures
- Types of tapes

Sources of Errors in Taping

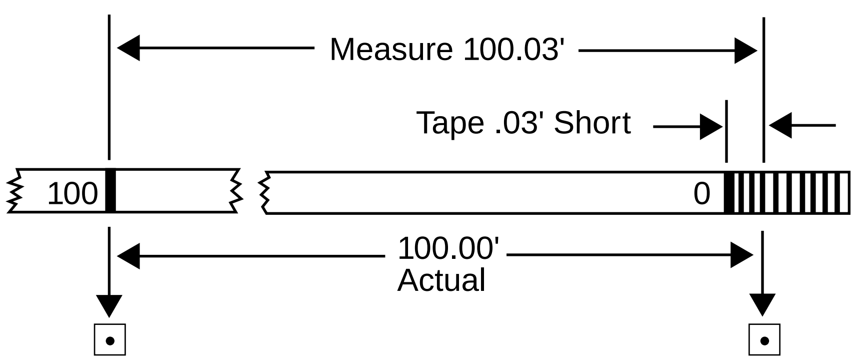


Figure 3-3. *Measuring with a short tape.*

- Incorrect length of tape

  $C_\ell = \left(\frac{\ell - \ell'}{\ell'}\right) L'$

  $L = L' + C_\ell$

  Where:

  L = Length of line

  ℓ = Actual tape length

  ℓ' = Nominal tape length

  L' = Measured length of line

  C_ℓ = Correction to the measured length of line

- Temperature other than standard

  $L = L' + k(T' - T)L'$

  Where:

  L = Length of line

  L' = Measured (recorded) length of line

  k = Coefficient of thermal expansion and contraction of the tape

  T' = Tape temperature at the time of measurement

  T = Standardized tape temperature

Temperature Correction Problem

A field crew uses a tape that has been standardized at 68° F in the field at a temperature of 45° F. What is the corrected distance if the observed distance is 850.44 ft?

- Tension other than standard

  $L = L' + (P' - P)\ \frac{L'}{A\,E}$

  Where:

  L = Length of line

  L' = Measured (recorded) length of line

  P' = Tension applied to tape in lb

  P = Standardized tension of tape in lb

  A = Cross-sectional area of tape in square in

  E = Modulus of elasticity of steel in lb per square in

Tension Correction Problem

A field crew uses a tape that has been standardized at a tension of 12 lb. In the field, the tape is used at a tension of 20 lb to measure a distance of 650.45 ft. What is the corrected distance?

- Sag

$$C_s = -\frac{W^2\ (L_s)}{24\ (P')^2} = \frac{w^2\ L_s^3}{24(P')^2}$$

$$L = L' + \sum C_s$$

Where:

L = Length of line

L' = Measured (recorded) length of line

C_s = Correction for sag

W = Total weight of tape between supports in lb

w = Weight of tape per ft in lb

L_s = Unsupported length of tape

P' = Tension in lb

Sag Correction Problem

A field crew uses a 100' tape weighing 1.5 lb that has been standardized for tension on a flat surface. In the field, the crew uses the tape supported at the ends only at a tension of 12 lb and measures a distance of 350.42 ft. What is the corrected distance?

- Poor alignment

$$C_A = -\frac{d^2}{ZL_T}$$

Where:

d = Distance tape off line

L_T = Length of tape

$L = L' + \sum C_A$

L = Actual length of line

L' = Measured length of line

$\sum C_A$ = Sum of alignment corrections for line

Poor Alignment Correction Problem

When a field crew measures a distance AB, the end of the first tape length is offline by 1.26 ft. The end of the second tape length is offline by an additional 0.98 ft. The end of the third tape length is offline by an additional 2.54 ft and the end of the final tape length is offline by an additional 1.66 ft. What is the correction for poor alignment if the measured distance is 350.56 ft? Assume four taped lengths, the first three are 100 ft each.

Corrections when measuring between points

Corrections when laying out points

Distance Measurement by EDM

- Fundamental principle of EDM operation
- Reduction of slope distance to horizontal distance

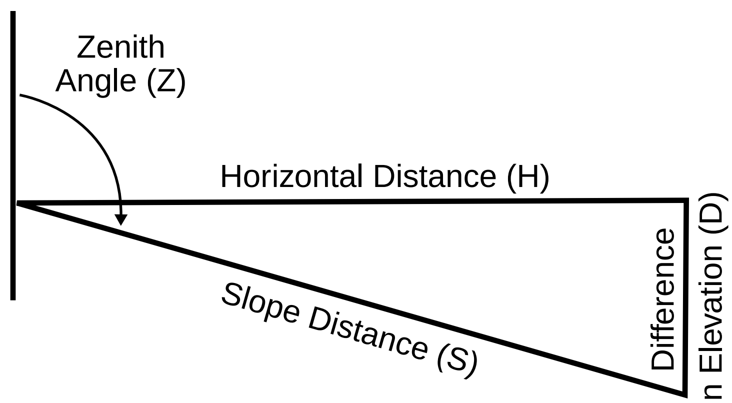


Figure 3-4. *Slope distance reduction.*

For measured Zenith angle:

$H = S \sin Z$

For known difference in elevation:

$H = \sqrt{S^2 - d^2}$

Sources of Errors in EDM Measurement

- Standard error of the instrument
- Temperature, pressure and humidity

Elevation Measurements

Error Sources

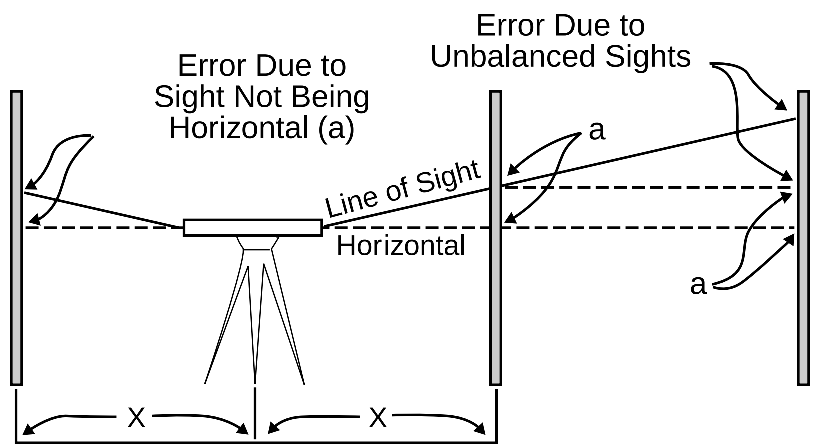


Figure 3-5. *Error due to unbalanced sights.*

Angular (Direction) Measurements

Introduction

- Horizontal vs. vertical
- Three basic requirements to determine angle
- Units of measure

Bearings and Azimuths

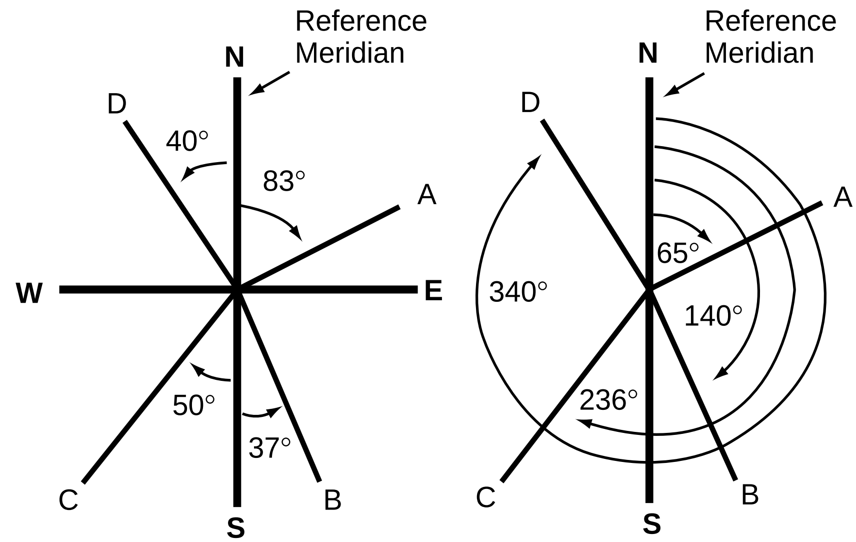


Figure 3-6. *Measurement of bearings and azimuths.*

Magnetic Declination and Bearings

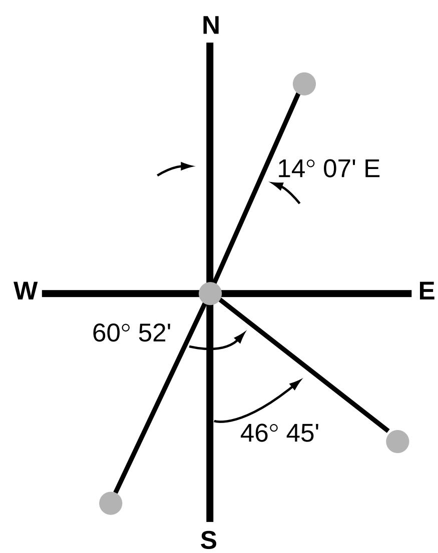


Figure 3-7. *Relationship between magnetic and astronomic bearings.*

Equipment Types

Basic Theodolite/Transit Operations

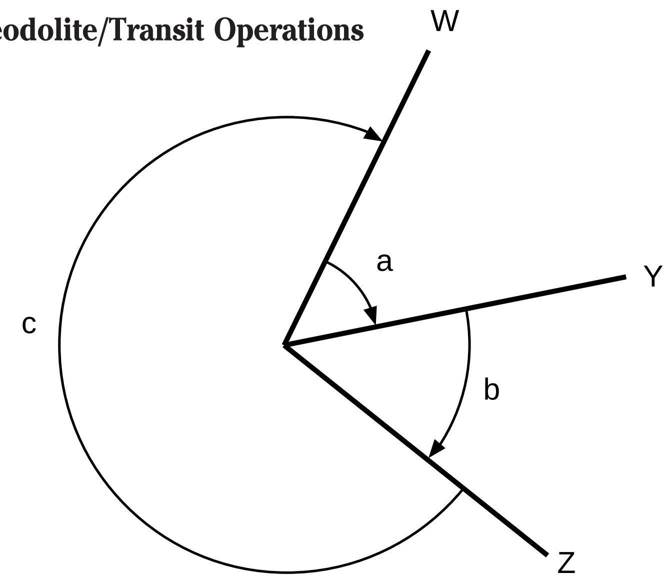


Figure 3-8. *Closing the horizon.*

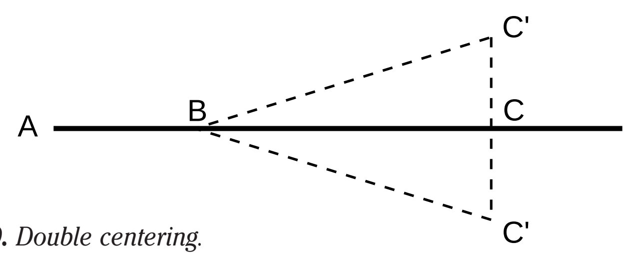


Figure 3-9. *Double centering.*

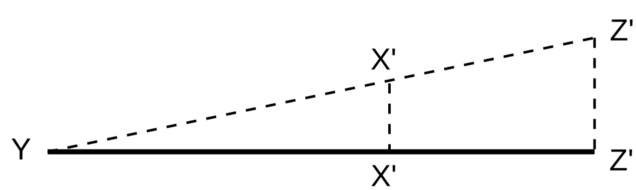
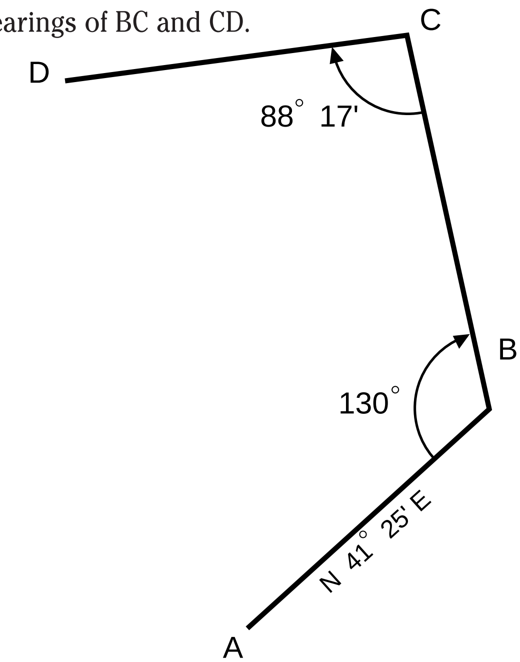


Figure 3-10. *Balancing in.*

Example Problems

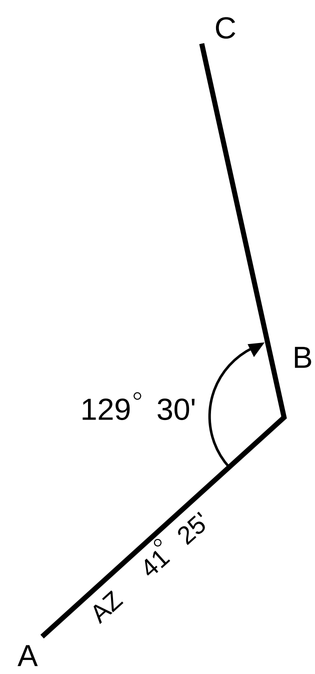
Problem 1.

Determine the bearings of BC and CD.



Problem 2.

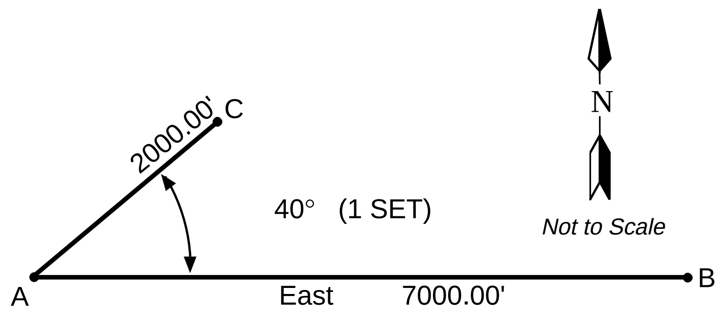
Determine the azimuth of BC.



Sample Test Questions

1. A-4 1991 LS

The measurements to Point C are shown below. A 6" theodolite with a measured standard error of ± 20" per angle set (direct and reverse) from all sources and an EDM with a standard error of ± .02 + 5 PPM was used.



Required:

A. Compute and sketch the standard error ellipse for point C. Label and dimension the semimajor axis, semiminor axis, and the orientation angle, U. Assume uncorrelated measurements.

B. What is the probability that the measured point is within or on the standard ellipse?

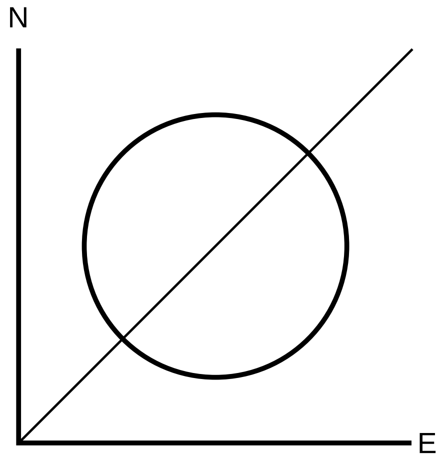
C. What are the dimensions of the semimajor and semiminor axis if probability of 95% is wanted?

D. What is the minimum number of angle sets needed to decrease the semimajor axis to 0.12' on the standard error ellipse?

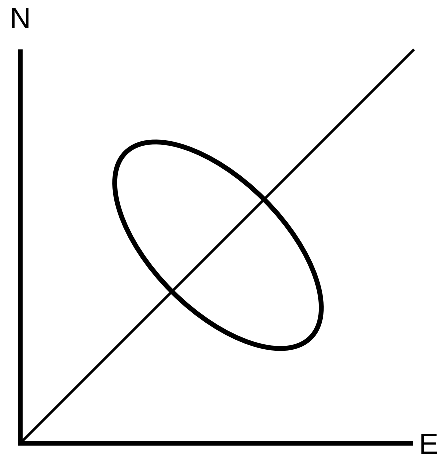
E. Each of the error ellipses shown on the following page indicates the relative comparative accuracy of establishing a point location with one of the following instrument combinations, A through F. In the spaces provided in the solution booklet, indicate the letter corresponding to the instrument combination that best works with the ellipses.

Angle	Angle Measured With:	Distance Measured With:
A	Transit	EDM
B	Compass	Steel Tape
C	Theodolite	EDM
D	Compass	Gunter's Chain
E	Transit	Steel Tape
F	Theodolite	Steel Tape

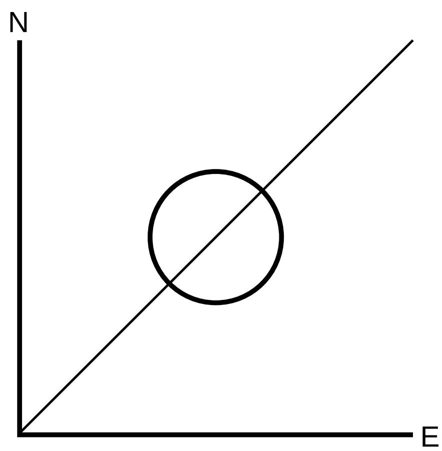
Assume the line is 2000' at an azimuth of 45°.



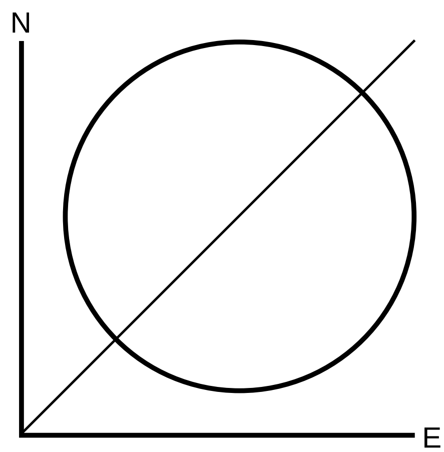
1. _____



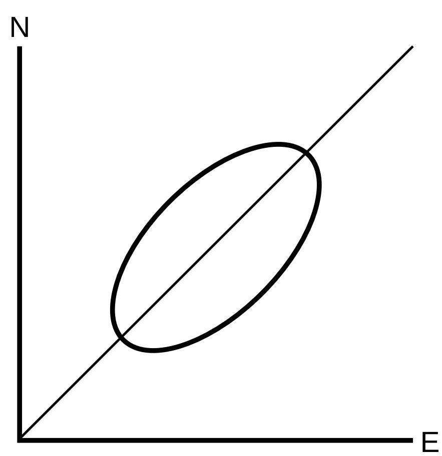
2. _____



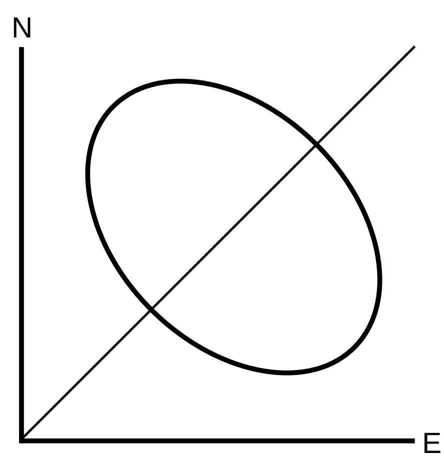
3. _____



4. _____



5. _____



6. _____

2. A steel tape standardized at 68° F. and supported throughout under a tension of 12 lb was found to be 99.991 ft long. This tape was used to lay out a horizontal distance AB of 650.23 ft. The ground was on a smooth 4% grade; thus, the tape was used fully supported. Determine the slope distance if a pull of 25 lb was used and the temperature was 42° F. (The cross sectional area of the tape = 0.005 sq. in. and the elasticity of steel = 29,000,000 lb/sq. in)

3. From the bearings below, compute the azimuths (measured from the North) and determine the angle, measured clockwise, between successive bearings.

   N 73° 34' 22"E, S 32° 15' 45"E, N 32° 12' 24"E, N 89° 59' 17"W.

4. A line AB is prolonged to point C by double centering. Two foresight points C' and C" are set. What is the the angular error introduced in a single plunging if the length BC is 900.44 ft and C' C" is 0.67 ft?

Answer Key

1. A. Semiminor axis = error in distance

$$= \pm\ .02 + \left(\frac{2000}{1000000}\ (5)\right)$$

$$= \pm 0.03'$$

or

$$= \pm \sqrt{(.02)^2 + \left(\frac{2000}{1000000}\ (5)\right)^2}$$

$$= \pm\ 0.022$$

Semimajor axis = error in angle

$$= \frac{20''}{206265} \text{ x } 2000$$

$$= \pm\ 0.194$$

Orientation angle (U) is angle between semimajor axis and the x or easting axis

$$\therefore 40° + 90° = 130°$$

B. Probability P [U≤ C^2] is represented by the volume under the bivariate normal density surface within the region defined by the error ellipse. For the standard error ellipse C 1, the probability is 0.394 or 39.4%.

C. For $P[U \leq C^2] = 0.95$, $C = 2.447$

= data from published tables

$\therefore$ semimajor axis = $0.194 \times 2.447 = 0.475$

or = $0.030 \times 2.447 = 0.073$

semiminor axis = $0.022 \times 2.447 = 0.054$

D. Number of angle sets needed = $\dfrac{0.194}{\sqrt{n}} = 0.12$

$n = 2.61$, therefore 3 sets

E.

Ellipse No.	Instrument Combination
1	E
2	A
3	C
4	D
5	F
6	B

2. True slope distance to be laid out:

4% of 650.23' = 26.009' (which equals the vertical distance)

Slope distance = $\sqrt{650.23^2 + 26.009^2}$

= 650.75'

Correction for incorrect length:

((99.991 - 100) / 100) 650.75 = -0.059'

Correction for temperature:

(0.0000065 (42-68)) 650.75 = -0.110'

Correction for tension:

(25-12) (650.75 / (.005 (29,000,000)) = 0.058

Slope distance to lay out:

650.75 - (-0.059) - (-0.110) - 0.058 = 650.86'

3. Bearings to azimuths:
   N 73° 34' 22"E = 73° 34' 22"
   S 32° 15' 45"E = 180° - 32° 15' 45" = 147° 44' 15"
   N 32° 12' 24"E = 32° 12' 24"
   N 89° 59' 17"W = 360° - 89° 59' 17" = 270° 00' 43"

   Clockwise angles:
   147° 44' 15" - 73° 34' 22" = 74° 09' 53"
   360° + 32° 12' 24" - 147° 44' 15" = 244° 28' 09"
   270° 00' 43" - 32° 12' 24" = 237° 48' 19"

4. 0.69' / 2 = 0.345' = The error in a single plunging
   Tangent of the angle = 0.345 / 900.44
   Tangent of the angle = 0.0003831
   Angle = 0° 01' 19"

References

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