UNIT 2

# BASIC SURVEY MATH

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## Introduction

The purpose of this video unit is to present basic math concepts and principles useful to survey computations. It has been assumed that most viewers are already familiar with some or most of the topics presented in the beginning of the unit. It is important to have a developed understanding of the basic operations of arithmetic, algebra, geometry, and trigonometry. This unit is not designed as a complete math course, but rather as an overview and guide to computation processes unique to surveying and mapping. Surveyors who need to work on math operations and fundamental skills addressed in the video will find sources for further study in the reference section at the end of this unit.

Survey mathematics generally consists of applications of formulas and equations that have been adapted to work toward the specific needs of the surveyor such as:

- Units of measurement and conversions
- Check and adjustment of raw field data
- Closure and adjustment of survey figures
- Calculations for missing elements of a figure
- Working with coordinates (COGO)
- Intersections of straight lines and of circles

It is hoped this video unit will help viewers to recognize solution formats for problems and then make correct and effective use of appropriate methods to solve these particular survey problems.

## Performance Expected on the Exams

Recognize solution formats, and make correct and effective use of appropriate mathematical solutions to particular survey applications.

## Key Terms

Absolute value
Adjacent side
Algebra
Arc
Arithmetic
Azimuth
Bearing
Central angle
Chord
Circular curve
Circumference
Complementary angle
Coordinate conversion
Cosecant
Cosine
Cotangent
Cubes
Decimal system
delta x, delta x
Departure
External distance
Geodetic north
Grads
Grid north

Hexagon
Hypotenuse
Intersection of straight line and arc
Inverse processes
Law of cosines
Length of arc
Meter
Most probable value
Opposite side
Parabola
Pentagon
Percentage
Plane geometry
Polygon
Quadrants
Quadrilateral
Radius
Random error
Rectangular coordinates
Rhomboid
Roots
Sag curve
Sector of a circle
Sexagesimal system
Significant figures
Sine
Squares
Supplementary angles
Tangent
Horizontal curve
Intersections
Intersections of straight lines
Latitude
Law of sines
Magnetic north
Mid-ordinate distance
Oblique triangle
Order of operations
Parallelogram
Percent of slope
pi
Polar coordinates
Pythagorean theorem
Quadratic equation
Radian
Radius point
Rate of change
Residual
Right triangle
Rounding off
Secant
Segment of a circle
Signed numbers
Simultaneous equation
Square root
Standard error
US survey foot
Trigonometry

# Video Presentation Outline

## Arithmetic

- Decimal system
- Rounding off and significant figures
- Percentage
- Squares, cubes and roots

## Conversion of Units of Measure

- Converting lineal units
- Converting angular units
- Converting units of area

## Random Error Analysis

- Error definitions
- Error residuals
- Statistical error matrix
- Propagation of error
- Error in summation
- Error in product
- Error in series

## Algebra

- Signed numbers
- Equations
- Order of operations
- Parentheses
- Evaluating equations and combining terms
- Solving equations
- The quadratic equation formula

## Plane Geometry

- Angles
- Geometrical theorems
- Geometrical figures
- Polygons
- Triangles

## Trigonometry

- Right triangles
- Pythagorean theorem
- Trigonometric functions
- Oblique triangles
- Directions: bearings and azimuths
- Latitudes and departures
- Plane coordinates

## Coordinate Geometry

- Intersection of straight lines
- Intersection of straight line and arc
- Intersection of two arcs

# Sample Test Questions

1. The product of 416.78 multiplied by 210.98 is?

   A. 879.32
   B. 8,793.32
   C. 87,932.24
   D. 879,322.44

2. The quotient of 36.11 divided by 191.67 is?

   A. 188.40
   B. 18.84
   C. 1.88
   D. 0.19

3. Square the number 0.713729, showing the results to the nearest five decimal places.

   A. 0.50941
   B. 0.50940
   C. 0.50942
   D. 0.50943

4. The percentage of slope for a proposed ramp is +3.55%. What is the change in elevation of this ramp for a horizontal length of 356 ft?

   A. -126.38 ft
   B. +12.60 ft
   C. +12.64 ft
   D. +126.38 ft

5. Where the centerline slope of a highway has a vertical drop of 14.75 ft in 265 ft horizontally, what is the rate of change expressed in percentage?
   A. 0.55%
   B. 0.56%
   C. 5.55%
   D. 5.57%

6. Determine the square root of 0.6935, showing the result to the nearest five decimal places.

   A. 0.832776
   B. 0.83276
   C. 0.83277
   D. 0.832766

7. 24.91 expressed in ft and in, equals:

   A. 24 ft, 10-7/8 in
   B. 24 ft, 10-3/8 in
   C. 24 ft, 10-1/4 in
   D. 24 ft, 11 in

8. 4,178.309 meters equals _____ United States survey ft.

   A. 1,273.56 survey ft
   B. 1,273.55 survey ft
   C. 13,708.20 survey ft
   D. 13,708.34 survey ft

9. 6,172.98 United States survey ft equals _____ meters.

   A. 1,881.528 m
   B. 1,881.547 m
   C. 20,252.313 m
   D. 20,252.519 m

10. When converted to survey ft, 3,421.381 meters equals _____ survey ft.

    A. 1,042.84 survey ft
    B. 1,042.85 survey ft
    C. 11,224.87 survey ft
    D. 11,224.98 survey ft

11. 21.56 chains converts to _____ survey ft.

    A. 1,422.36 survey ft
    B. 1,386.00 survey ft
    C. 1293.60 survey ft
    D. 1,422.96 survey ft

12. 14° 34' 37" converted to radian measurement is _____?

    A. 0.250345 rad
    B. 0.254416 rad
    C. 0.250351 rad
    D. 0.250337 rad

13. 0.758612 rad, when converted to degrees, minutes, and seconds is _____.

    A. 43° 46' 52"
    B. 43° 41' 35"
    C. 43° 33' 12"
    D. 43° 27' 55"

14. How many hectares are contained in a rectangular parcel that measures 19.23 ch. x 40.63 ch.?

    A. 78.131 hec
    B. 781.315 hec
    C. 31.619 hec
    D. 193.063 hec

15. An angle has been measured six individual times with the following results: a.) 46° 21' 45"; b.) 46° 22' 10"; c.) 46° 22' 05"; d.) 46° 22' 00"; e.) 46° 21' 45"; f.) 46° 21' 55". What is the most probable value of the angle?

    A. 46° 21' 45"
    B. 46° 21' 50"
    C. 46° 21' 57"
    D. 46° 22' 00"

16. Determine the standard error for the following group of six measurements: a.) 11,249.71 ft; b.) 11,250.06 ft; c.) 11,249.86 ft; d.) 11,249.99 ft; e.) 11,250.01 ft; f.) 11,249.98 ft.

    A. ±0.13 ft
    B. ±0.12 ft
    C. ±0.10 ft
    D. ±0.08 ft

17. Determine the standard error of the mean for the measurement set in problem #16.

    A. ±0.21 ft
    B. ±0.13 ft
    C. ±0.05 ft
    D. ±0.03 ft

18. A rectangular parcel of land was surveyed. The measurement for side X was 339.21 ft with an error of ±0.05 ft. Side Y was measured as 563.67 ft, with an error of ±0.09 ft. What is the area of the parcel and what is the expected error in the area?

    A. Area = 191,202 $ft^2$ or 4.389 ac.; standard error = ±41.7 $ft^2$
    B. Area = 191,202 $ft^2$ or 4.389 ac.; standard error = ±41.5 $ft^2$
    C. Area = 191,202 $ft^2$ or 4.389 ac.; standard error = ±24.1 $ft^2$
    D. Area = 191,202 $ft^2$ or 4.389 ac.; standard error = ±53.5 $ft^2$

19. The total length for a highway centerline was measured in four different segments using different equipment and different methods of measurement on different days. The total length of the line was found by totaling the length of each segment. Standard error for each segment was determined to be:

    Standard Error of Segment #1 =±0.04 ft
    Standard Error of Segment #2 =±0.03 ft
    Standard Error of Segment #3 =±0.08 ft
    Standard Error of Segment #4 =±0.11 ft

    The standard error of the total distance of the centerline is _____?

    A. Standard error of the sum = ±0.14 ft
    B. Standard error of the sum = ±0.26 ft
    C. Standard error of the sum = ±0.02 ft
    D. Standard error of the sum = ±0.07 ft

20. What is the sum of the following five numbers: (-230.67); (+517.39); (+100.26): (-311.47); and (-481.28)?

    A. 405.77
    B. 1,641.07
    C. -1,641.07
    D. -405.77

21. The remainder after -146.11 has been subtracted from -37.82 is _____ ?

    A. -108.29
    B. 108.29
    C. -183.93
    D. 183.93

22. Write an equation based on the following word statement: “three times a number, plus the number cubed, minus the number multiplied by 87.” In the algebraic equation, let b stand for the number referred to in the problem statement.

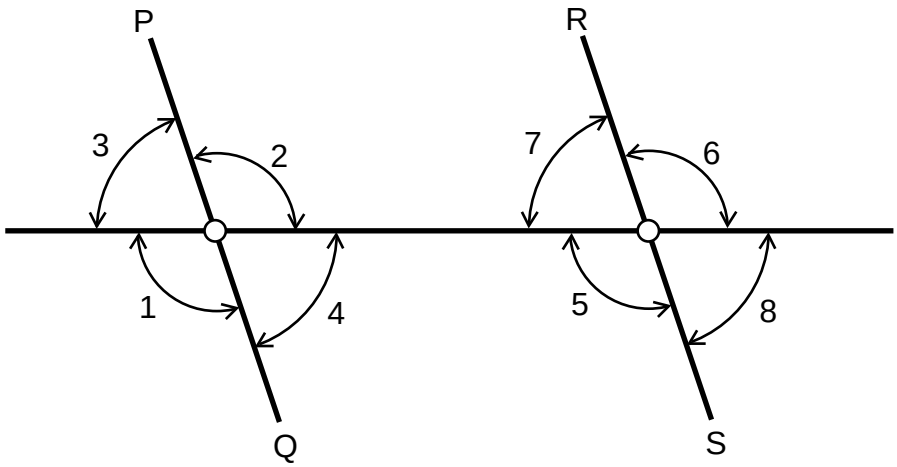
    A. $3 (b + b^{3)} - (87b)$
    B. $3 (b + b^3) - 87b$
    C. $3 b + b^3 - 87b$
    D. $(3b) + b^3 - (87b)$

23. Letting w = 12 and z = 3, evaluate the following equation: $5w + (21 - w) 14z + (z - 23)$.

    A. 418
    B. 2,878
    C. 2,881
    D. 19,162

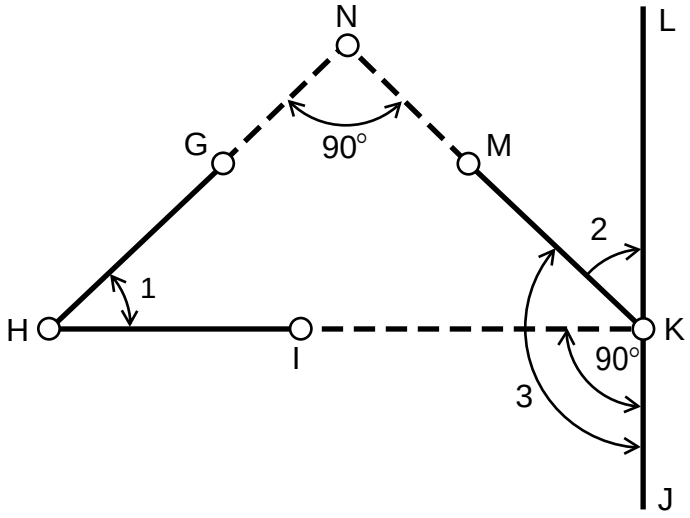
24. If angle 3 in the sketch below is 71° 39' 12", calculate the values of angles 1, 2, 5, and 8. Assume lines P-Q and R-S are parallel.

A. <1 = 108° 20' 48"; <2 = 108° 20' 48"; <5 = 108° 20' 48"; <8 = 71° 39' 12"
B. <1 = 108° 20' 48"; <2 = 71° 39' 12"; <5 = 71° 39' 12"; <8 = 108° 20' 48"
C. <1 = 108° 20' 48"; <2 = 71° 39' 12"; <5 = 108° 20' 48"; <8 = 71° 39' 12"
D. <1 = 108° 20' 48"; <2 = 108° 20' 48"; <5 = 71° 39' 12"; <8 = 71° 39' 12"




25. If angle 1 in the sketch below is 46° 11' 20", calculate the values of angles 2 and 3.

A. <2 = 46° 11' 20"; <3 = 133° 48' 40"
B. <2 = 43° 48' 40"; <3 = 133° 48' 40"
C. <2 = 46° 11' 20"; <3 = 136° 11' 20"
D. <2 = 43° 48' 40"; <3 = 136° 11' 20"

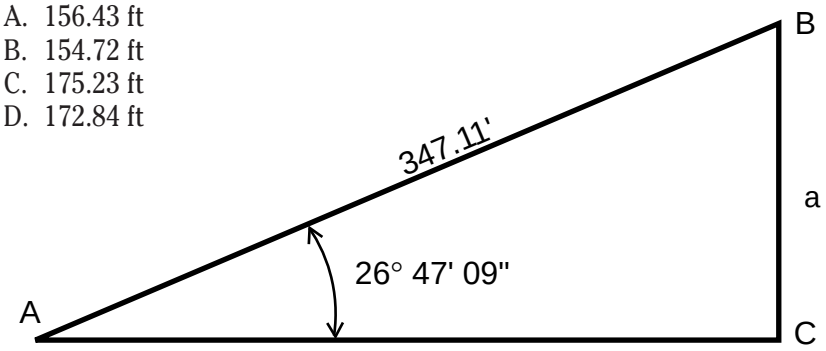


26. Solve for angle A in the triangle shown below:

A. 15° 38' 56"
B. 15° 23' 08"
C. 15° 58' 20"
D. 15° 58' 21"

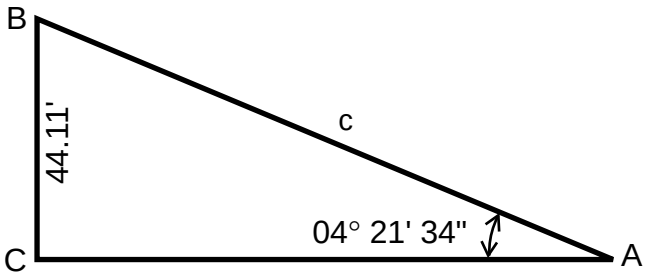

27. Solve for the missing side "a" of the triangle in the sketch below.

A. 156.43 ft
B. 154.72 ft
C. 175.23 ft
D. 172.84 ft



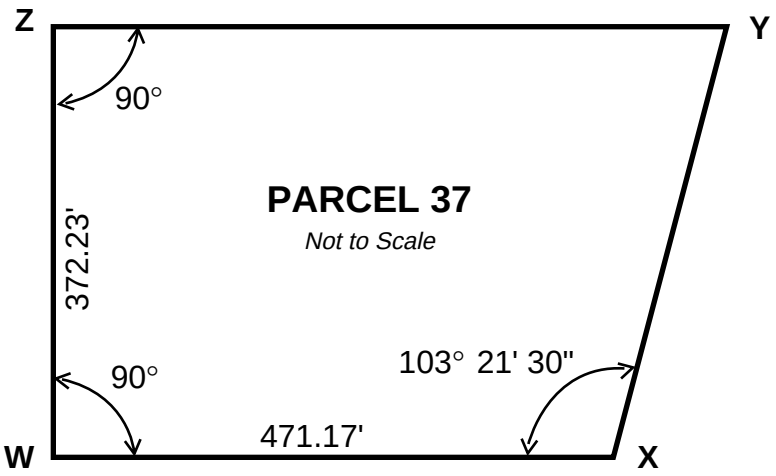

28. What is length of side "c" in the triangle shown in the sketch below?

A. 578.61 ft
B. 598.75 ft
C. 600.36 ft
D. 580.29 ft



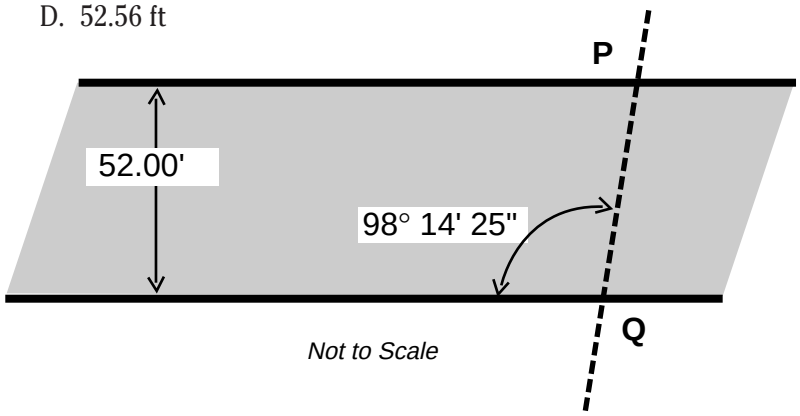
29. What is the length of side Y to Z in the sketch of parcel #37 shown below?

A. 557.17 ft
B. 559.56 ft
C. 558.56 ft
D. 556.25 ft



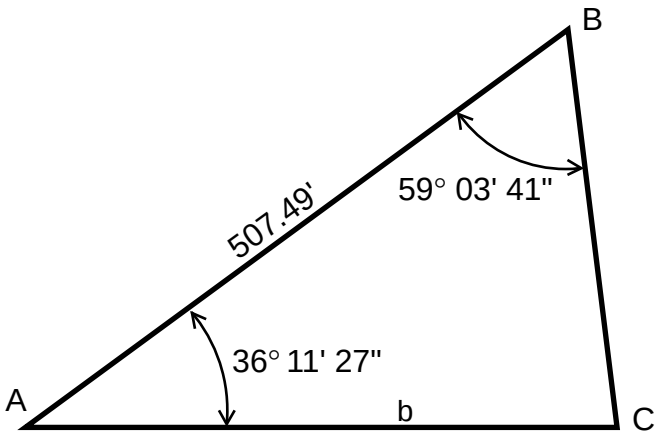

30. Line P to Q intersects the street alignment as shown in the sketch below. What is the length of line P-Q?

A. 52.50 ft
B. 52.52 ft
C. 52.54 ft
D. 52.56 ft



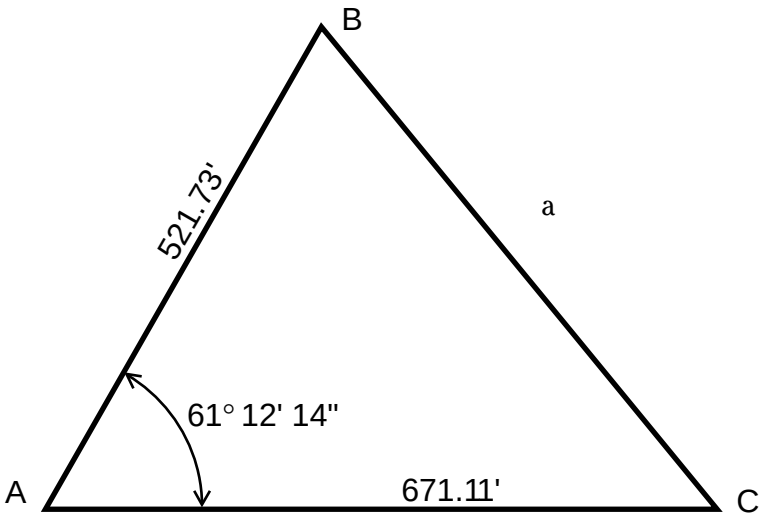
31. Given the following dimensions shown for the oblique triangle in the sketch below, solve for the length of side “b.”

A. 855.58 ft
B. 857.02 ft
C. 438.34 ft
D. 37.12 ft



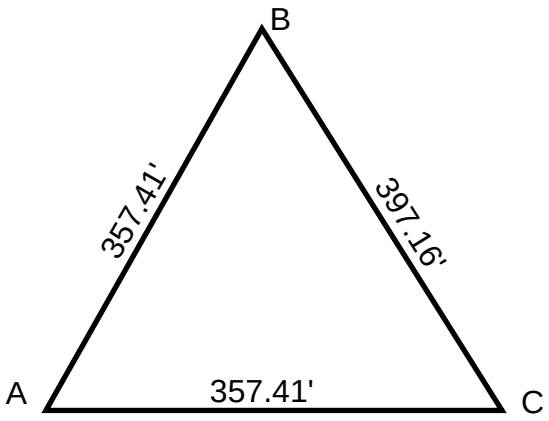

32. Solve for side “a” using the elements given for the oblique triangle shown in the sketch below.

A. 751.27 ft
B. 618.43 ft
C. 744.27 ft
D. 620.70 ft



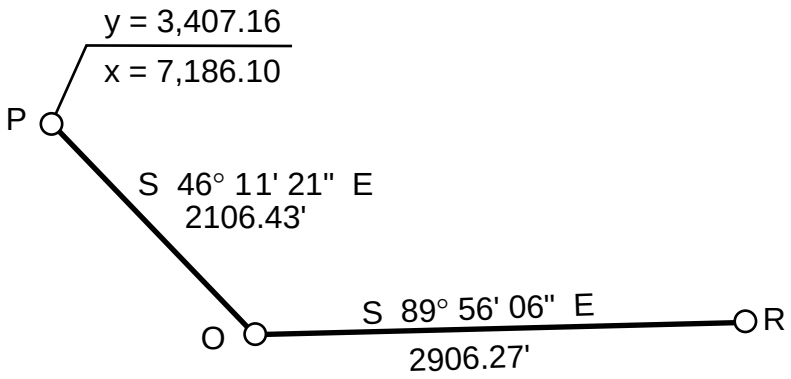
33. From the elements of the oblique triangle given in the sketch below, solve for angle A.

A. 68° 06' 10"
B. 67° 54' 41"
C. 67° 50' 53"
D. 67° 30' 19"



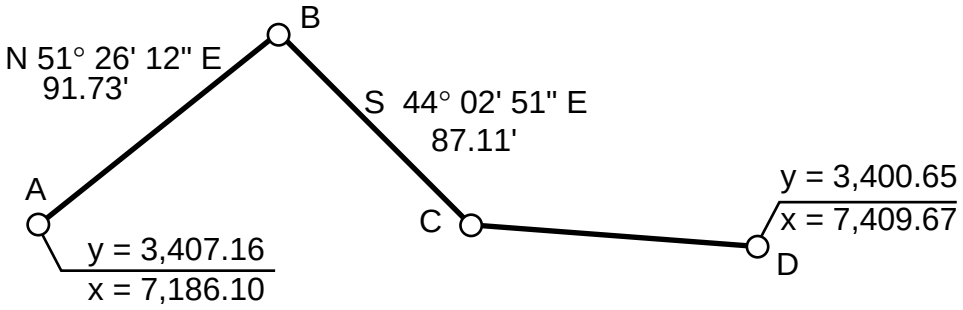

34. Using information given in the sketch below, calculate the coordinates for point R.

A. y = 1,945.62; x = 11,612.43
B. y = 1,945.73; x = 11,612.39
C. y = 1,945.61; x = 11,612.33
D. y = 1,945.68; x = 11,612.39



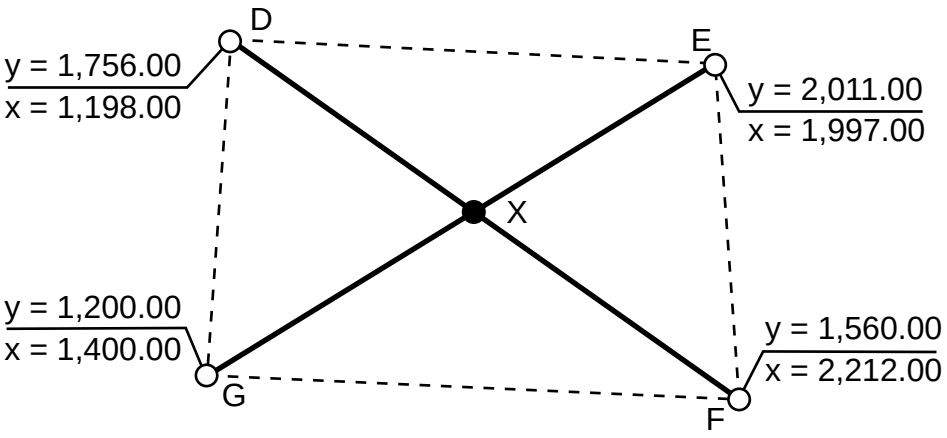
35. After looking at the sketch below, determine the bearing and distance of the line from point C to point D.

A. N 89° 19' 27" W; 91.33 ft
B. S 89° 49' 06" W; 91.31 ft
C. S 89° 19' 27" E; 91.33 ft
D. S 89° 19' 17" E; 91.29 ft



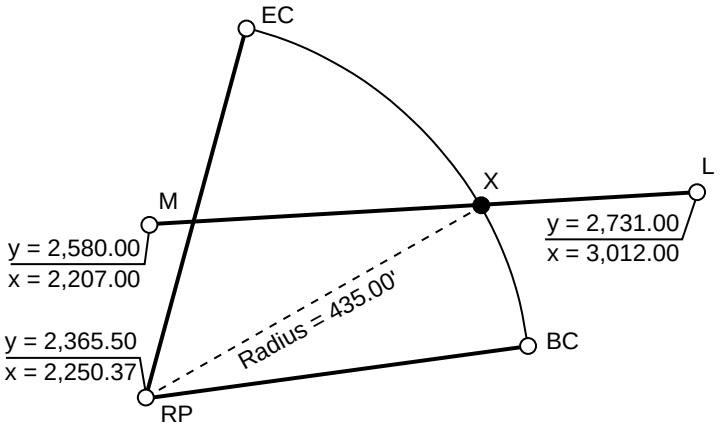

36. Determine the coordinates of the point "x"where the two lines intersect as shown in the sketch below.

A. y = 1,652.56; x = 1,733.14
B. y = 1,652.47; x = 1,732.99
C. y = 1,652.64; x = 1,733.24
D. y = 1,652.53; x = 1,733.19



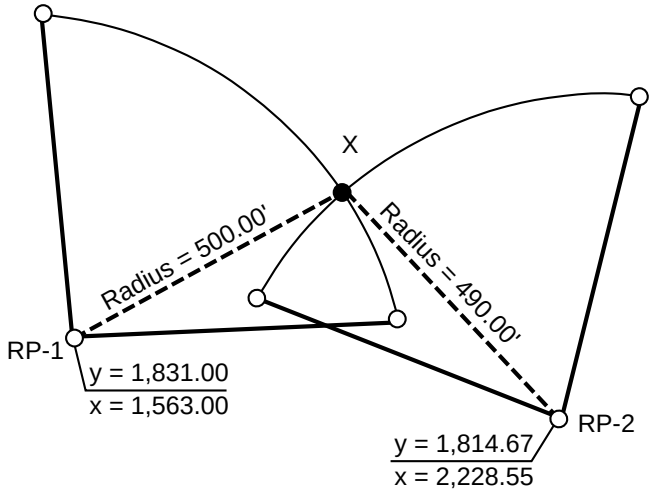
37. Using information given in the sketch, calculate the bearing of the line from the "RP" to point X. Also determine the distance from L to X.

    A. Bearing "RP" to X = N 48° 51' 10" E; Distance L to X= 438.71 ft
    B. Bearing "RP" to X = N 50° 51' 50" E; Distance L to X= 439.85 ft
    C. Bearing "RP" to X = N 49° 10' 30" E; Distance L to X= 440.00 ft
    D. Bearing "RP" to X = N 49° 12' 40" E; Distance L to X= 441.07 ft



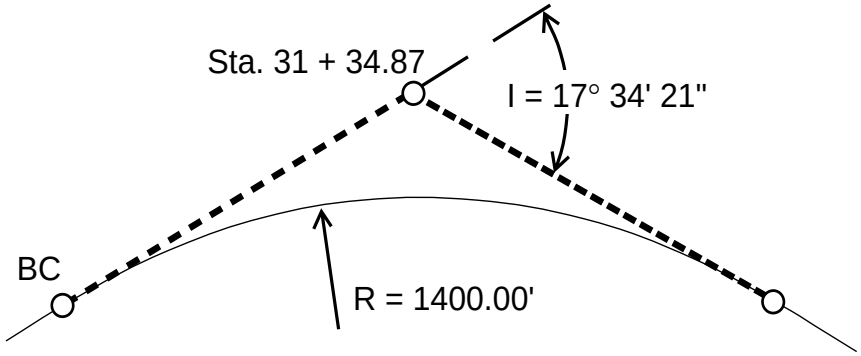

38. Determine the bearing of lines RP-1 to X and line RP-2 to X from the survey data shown in the sketch below.

    A. Bearing Line RP-1 to X = N 44° 17' 50" W;
       Bearing Line RP-2 to X = N 40° 12' 45" E
    B. Bearing Line RP-1 to X = 40° 12' 45" E;
       Bearing Line RP-2 to X = N 40° 12' 45" W
    C. Bearing Line RP-1 to X = N 44° 17' 50" E;
       Bearing Line RP-2 to X = N 40° 12' 45" W
    D. Bearing Line RP-1 to X = N 40° 12' 45" W;
       Bearing Line RP-2 to X = N 44° 17' 50" W



39. Determine the deflection angle and the sub-chord length (from beginning of curve) required to locate sta. 30+74.50 on its correct position on the arc using data given in the sketch below.

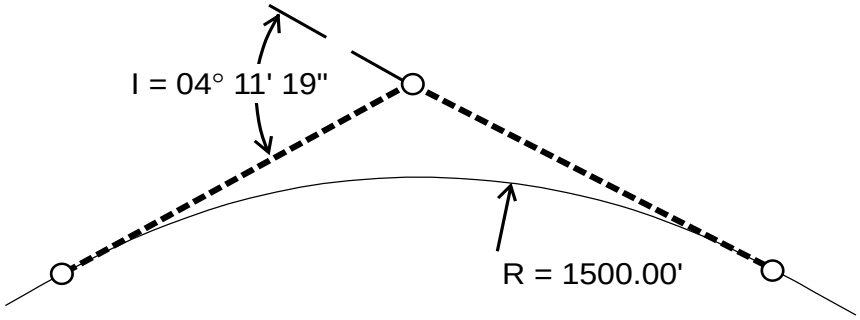
    A. Deflection = 03° 11' 33"; Sub-chord = 155.94 ft
    B. Deflection = 06° 23' 06"; Sub-chord = 155.69 ft
    C. Deflection = 06° 23' 06"; Sub-chord = 311.48 ft
    D. Deflection = 03° 11' 33"; Sub-chord = 152.07 ft




*Not to Scale*

40. Using curve information given in the sketch, calculate the external and mid-ordinate distances for the curve.

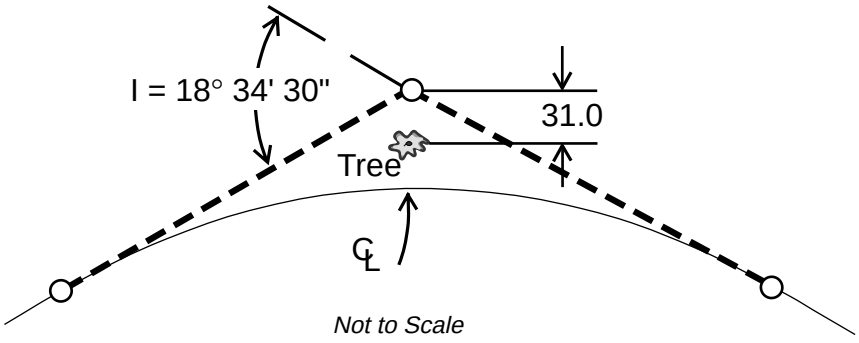
    A. External = 0.37 ft; Mid-ordinate = 0.37 ft
    B. External = 1.00 ft; Mid-ordinate = 1.00 ft
    C. External = 1.04 ft; Mid-ordinate = 1.04 ft
    D. External = 4.15 ft; Mid-ordinate = 4.14 ft




*Not to Scale*

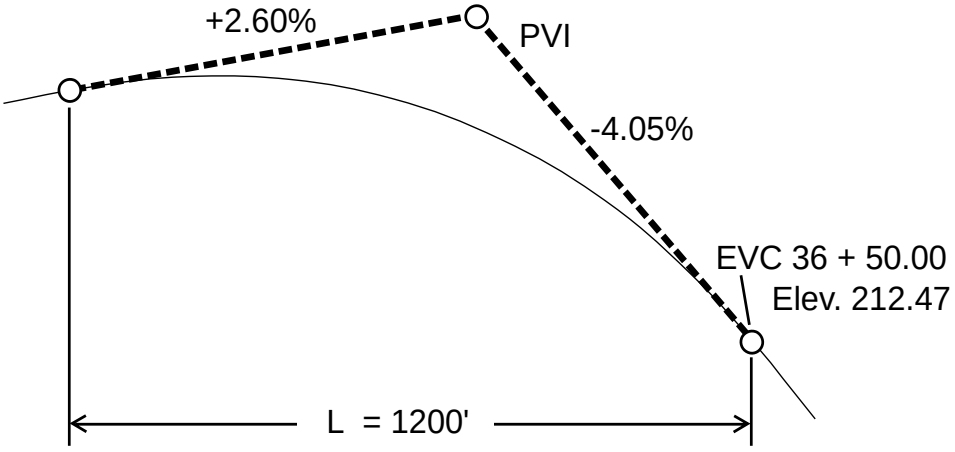
41. Using the data given in the sketch below, calculate the centerline radius that will allow the outside edge of a 42-ft roadway (overall width) to clear the center of the tree by 6 ft.

    A. C/L radius = 4,366.42 ft
    B. C/L radius = 4,478.80 ft
    C. C/L radius = 4,424.92 ft
    D. C/L radius = 4,436.80 ft




42. From data given on the sketch of the vertical curve below, calculate the elevation at station 31+56.

    A. Elev. @ sta. 31+56 = 225.02 ft
    B. Elev. @ sta. 31+56 = 225.25 ft
    C. Elev. @ sta. 31+56 = 225.47 ft
    D. Elev. @ sta. 31+56 = 225.72 ft

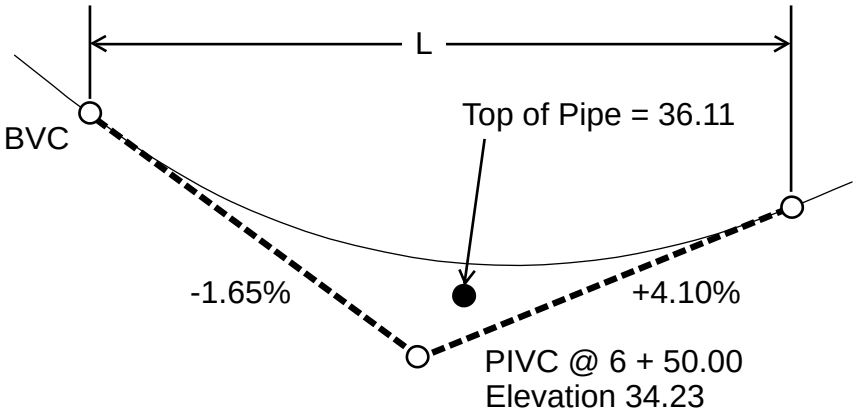


43. Referring back to Problem #42 (above), calculate the station and elevation of the high point of the curve.

    A. Sta. = 29+19; Elev. = 205.74 ft
    B. Sta. = 29+19; Elev. = 227.27 ft
    C. Sta. = 30+50; Elev. = 226.80 ft
    D. Sta. = 33+88; Elev. = 221.17 ft

44. Using the information given in the diagram below, calculate the station and elevation of the BVC of curve designed to provide a minimum of 3.0 ft clearance at the top of pipe located at station 6+87. Determine "L" to the nearest half station.

    A. Sta. @ BVC = 3+50; Elev. = 39.18 ft
    B. Sta. @ BVC = 4+00; Elev. = 38.36 ft
    C. Sta. @ BVC = 4+25; Elev. = 37.94 ft
    D. Sta. @ BVC = 3+25; Elev. = 39.59 ft



# Answer Key

1. C. 87,932.24
2. D. 0.19
3. A. 0.50941
4. C. +12.64 ft
5. D. -5.57%
6. C. 0.83277
7. A. 24 ft – 10 7/8 in
8. D. 13,708.34 survey ft
9. A. 1,881.528 meters
10. D. 11,224.98 survey ft
11. D. 1,422.96 survey ft
12. B. 0.254416 rad
13. D. 43° 27' 55"
14. C. 31.619 hec
15. C. 46° 21' 57"
16. B. ±0.12 ft
17. C. ±0.05 ft
18. B. 191,202 $ft^2$ or 4.389 ac.; std. error = ±41.5 $ft^2$
19. A. ±0.14 ft
20. D. -405.77
21. B. 108.29
22. D. ~~3b – 87b~~
23. A. 418
24. A. <1 = 108° 20' 48"; <2 = 108° 20' 48"; <5 = 108° 20' 48"; <8 = 71° 39' 12"
25. A. <2 = 46° 11' 20"; <3 = 133° 48' 40"
26. B. 15° 23' 08"
27. A. 156.43 ft
28. D. 580.29 ft
29. B. 559.56 ft
30. C. 52.54 ft
31. D. 437.12 ft
32. D. 620.70 ft
33. D. 67° 30' 19"
34. A. y = 1,945.62; x = 11,612.43
35. D. S 89° 19' 17" E; 91.29 ft
36. A. y = 1,652.56; x = 1,733.14
37. C. Bearing “RP” to X = N 49° 10' 30" E; Distance L to X= 440.00 ft
38. C. Bearing Line RP-1 to X = N 44° 17' 50" E;
    Bearing Line RP-2 to X = N 40° 12' 45" W
39. A. Deflection = 03° 11' 33"; Sub-chord = 155.94 ft
40. C. External = 1.00 ft; Mid-ordinate = 1.00 ft
41. A. C/L radius = 4,366.42 ft
42. D. Elev. @ sta. 31+56 = 225.72 ft
43. B. Sta. = 29+19; Elev. = 227.27 ft
44. D. Sta. @ BVC = 3+25; Elev. = 39.59 ft

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